

Skoltech 2021

Differential Geometry of connections

30.10.2020

Midterm test problems

Please send solutions to *kzarian@mccme.ru* by 12:00 p.m. 5th November 2021

Consider the surface in the Euclidean space \mathbb{R}^3 given parametrically by the equations

$$x = u - \frac{u^3}{3} + uv^2, \quad y = v - \frac{v^3}{3} + vu^2, \quad z = u^2 - v^2.$$

1. Compute the first and the second quadratic forms g and h , the principal curvatures k_1 and k_2 , the Gaussian curvature $k_1 k_2$ and the mean curvature $\frac{1}{2}(k_1 + k_2)$.
2. Compute the curvature of the metric on the surface and compare it with the Gaussian curvature.
3. Compute the area of the domain on the surface defined by the inequality $u^2 + v^2 \leq 1$.
4. Compute the transformation of parallel transport on the tangent plane along the circle $u^2 + v^2 = 1$
 - (a) by solving the differential equation of parallel transport;
 - (b) by applying the Gauss-Bonnet formula.Compare the results of computations.