

<b>Course Title (in English)</b>	Vertex operator algebras
<b>Course Title (in Russian)</b>	Вертекс операторные алгебры

<b>Lead Instructor(s)</b>	Feygin, Evgeny
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### Course Description

Infinite-dimensional Lie algebras (such as Virasoro algebra or affine Kac-Moody algebras) turn out to be very important in various areas of modern mathematics and mathematical physics. In particular, they are very useful in the description of some field theories. In this context one arranges infinite number of the Lie algebra elements into a single object called field. This idea generalizes to the general theory of vertex operator algebras. VOAs capture the main properties of the infinite dimensional Lie algebras and have rich additional structure. Vertex operator algebras proved to be very useful in many situations; the classical example is the KP integrable hierarchy. They are also extensively used in modern algebraic geometry. Our goal is to give an introduction to the theory of vertex operator algebras from the modern mathematical point of view. We describe the main definitions, constructions and applications of the theory. The course is aimed at PhD students and master students.

<b>Course Prerequisites / Recommendations</b>	Basic Lie theory, the theory of affine Kac-Moody algebras
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<b>Аннотация</b>	
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Бесконечномерные алгебры Ли (такие как алгебра Вирасоро и аффинные алгебры Каца-Мууди) играют важную роль в различных областях математики и математической физики. В частности, они оказываются очень полезны при описании квантовых теорий поля. В частности, квантовые поля описываются как производящие функции бесконечного числа элементов подлежащей алгебры Ли. Этот подход обобщается и формализуется в рамках теории вертекс операторных алгебр. Структурная теория ВОА включает в себя основные конструкции теории бесконечномерных алгебр Ли, но также включает в себя ряд дополнительных важных понятий. Вертекс операторные алгебры оказались очень полезны в ряде классических задач; классическим примером является интегрируемая иерархия уравнений КП. Кроме того, теория ВОА используется в современной алгебраической геометрии. В нашем курсе мы планируем дать введение в теорию вертекс операторных алгебр с точки зрения современной математики. Мы дадим основные определения, сформулируем основные свойства и опишем ряд приложений теории. Курс рассчитан на аспирантов, магистрантов и сильных студентов старших курсов бакалавриата.

**Course Academic Level** PhD-level suitable for MSc students

**Number of ECTS credits** 6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Heisenberg algebras and Fock modules	Representation theory of the Heisenberg algebra	2	8	
Virasoro algebras, Verma modules	Representation theory of the Virasoro algebra	1	8	
Boson-fermion correspondence, Schur polynomials, KP hierarchy	Introduction to the boson-fermion correspondence	2	8	
Vertex operator algebras and Lie algebras	Definitions and basic examples of vertex operator algebras	3	8	
Representation theory of vertex operator algebras	Properties of VOAs and of representations of VOAs	3	8	

Assignment Type	Assignment Summary
Problem Set	Problems of the Heisenberg and Virasoro VOAs.
Problem Set	Problems on the operator product expansion of fields
Problem Set	Problems on the structure of modules over the vertex operator algebras.

Type of Assessment Graded

Grade Structure

Activity Type	Activity weight, %
Homework Assignments	30
Final Exam	50
Midterm Exam	20

A: 80

B: 70

C: 60

D: 50

E: 40

F: 0

Attendance Requirements Optional with Exceptions

Maximum Number of Students

	Maximum Number of Students
Overall:	20
Per Group (for seminars and labs):	20

Course Stream Science, Technology and Engineering (STE)

Course Term (in context of Academic Year) Term 3  
Term 4

Course Delivery Frequency Every two years

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Mathematical and Theoretical Physics	Mathematics and Mechanics

Course Tags Math  
Physics

Required Textbooks	ISBN-13 (or ISBN-10)
Kac V., Raina A., Rozhkovskaya N. Bombay lectures on Highest weight representations of infinite dimensional Lie algebras.	978-981-4522-19-9

Recommended Textbooks	ISBN-13 (or ISBN-10)
Frenkel E., Ben-Zwi D. Vertex algebras and algebraic curves	978-0-8218-3674-3

Knowledge
The structure theory and representation theory of vertex operator algebras

Skill
Mathematical language of modern quantum field theory.

Experience
Working with various problems of representation theory and mathematical physics.

Select Assignment 1 Type	Problem Set
Or Upload Example(s) of Assignment 1	<a href="https://ucarecdn.com/c72cd6fd-7022-42cf-bcb4-b84601053849/">https://ucarecdn.com/c72cd6fd-7022-42cf-bcb4-b84601053849/</a>
Assessment Criteria for Assignment 1	Proportional to the number of solved problems.
Select Assignment 2 Type	Problem Set
Or Upload Example(s) of Assignment 2	<a href="https://ucarecdn.com/982565ce-5863-4c74-b737-f97f68a66584/">https://ucarecdn.com/982565ce-5863-4c74-b737-f97f68a66584/</a>

**Assessment Criteria for  
Assignment 2**

Proportional to the number of solved problems.