1) КАК ИНТЕГРИРОВАТЬ

Тонкости компакт.

D-branes

\+ adjoint strong coupl.
Vafa Witten '94

2) qq-ХАРАКТЕРЫ

как характеры

отдельная их жизнь как статсуммы других инстантонных теорий

\+

CFT

3) defects

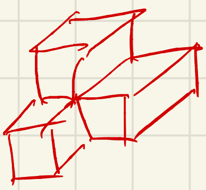
gauge origami

4) KZ, BPZ из 4х мерных теорий

формулы для волновых функций

беск-мерные представления алгебр

5) Higher dimensions



?

Некомпактные теории

Другие группы ?

open questions

$$\int \underbrace{\mathcal{D}A\,\mathcal{D}\psi\,\mathcal{D}\bar\psi\dots}\, \exp\left[-\frac{1}{g_0^2}\int_X \mathrm{Tr}\, F_A \wedge {*F_A} + \dots\right] \qquad \mathcal{O}_1 \dots \mathcal{O}_l$$

зависит только от Conf_X

X – 4-fold

$$= \sum_{k=0}^{\infty} q^k \int_{\mathcal{M}_k^+} \omega_1 \wedge \dots \wedge \omega_l$$

G - калибр. группа

$$k = k_P = -\frac{1}{8\pi^2}\int_X \mathrm{Tr}\, F_A \wedge F_A$$

пр-во модулей инстантонов

$$\{ A \mid F_A^+ = \tfrac{1}{2}(F_A + {*F_A}),\ A \text{ – связность в } P \text{ over } X \}$$

$q \in \mathbb{C}$

$\to$ инв. полином $\quad P_i \in (\mathrm{Sym}\,\mathfrak{g}^*)^G$

$$\mathcal{O}_i \sim P_i(\sigma)$$

$$\sigma \in \Gamma(X, \mathrm{ad}_P = \mathfrak{g}^{\mathbb{C}} \times_{\mathrm{Ad}} P)$$

↳ скаляр в векторном мультиплете

(часто используют ϕ вместо σ)

Локальная теория $X = \mathbb{R}^4$

$P = G \times X$

$$S_{YM} = \int_{\mathbb{R}^4} \mathrm{Tr}\, F_A \wedge * F_A < \infty$$

$$F_A = - * F_A$$

$\Longrightarrow$ Thm. (Uhlenbeck)

$P|_{D^4_\infty} = G \times D^4_\infty$

$P|_X = G \times X$

$$\boxed{S^4 = X \cup D^4_\infty}$$

$g^{-1} A g + g^{-1} dg$ продолжится ~~на~~ для некоторого g S^4

$\Downarrow$

$$\begin{array}{c} P \\ \downarrow \\ S^4 \end{array}$$

можно определить

$$k = -\frac{1}{8\pi^2} \int \mathrm{Tr}\, F_A \wedge F_A$$

$$\pi_3(G) = \mathbb{Z}$$

$g_\infty : S^3 \to G$

$$\frac{1}{8\pi^2} \int_{S^3} \mathrm{Tr}(g_\infty^{-1} dg_\infty)^3$$

Можно найти $\mathbb{C}$-решение уравнений Янга-Миллса $D_A^* F_A = 0$ с нечеткими S_{YM}

$$\mathcal{M}_k^{\text{framed}} = \{ A \mid F_A^+ = 0,\ A \text{ определено на } S^4 \} / \mathcal{G}_\infty$$

$$\mathcal{G}_\infty = \{ g : \mathbb{R}^4 \to G,\ g(x) \xrightarrow[x \to \infty]{} g_\infty = 1 \}$$

$$G \curvearrowright \mathcal{M}_k^{\text{framed}}$$

можно деформировать теорию, введя параметр. инфинитезимальные G-преобразования

Физически это соот. тому, что мы фиксируем

$$\sigma(x) \xrightarrow[x \to \infty]{} \sigma_\infty \text{ — фиксировано}$$

$G = U(N)$

$$\langle \quad \rangle_{\sigma_\infty = \mathrm{diag}(a_1,\dots,a_N)}$$

$$[\sigma_\infty, \bar\sigma_\infty] = 0$$

$\downarrow$

сохр. суперсимметрии

$$\sigma = \sigma_q + \sigma_\infty \qquad \sigma_q \to 0$$

$$S^{bos} = \underbrace{\|F_A^+\|^2_{L^2} + \|D_A\sigma\|^2_{L^2}}_{\|D_A\sigma_q\|^2_{L^2} + \|V_{\sigma_q}\|^2} + \|[\sigma_q,\bar\sigma_q]\|^2_{L^2} + S_{top}$$

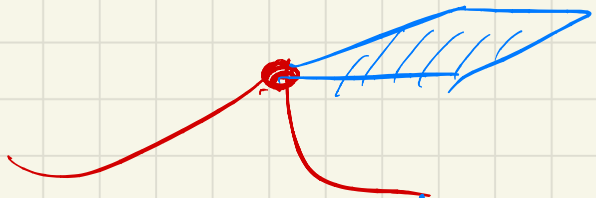
$$[F_A, \sigma_q] = 0$$

$$\Downarrow$$

$$D_A\sigma_q = 0 \Rightarrow \sigma_q = 0$$

и A - неприводима

Vector field on the moduli space M_k^{framed}

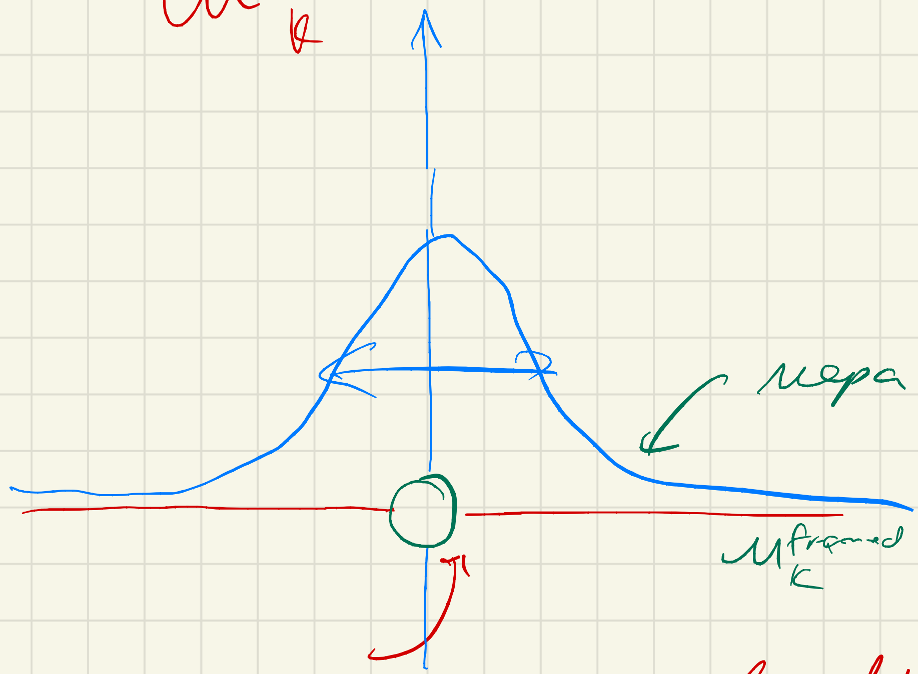


группа G свободно действует на M^{framed}

$$V_{\sigma_\infty} \neq 0$$

$$\int_{M_k^{framed}} \exp\left(-\|V_{\sigma_\infty}\|^2\right) \omega_1 \dots \omega_\ell$$

намек на эквивариантный интеграл



$Sym^k(\mathbb{R}^4)$ этой точка в M_k^{framed} нет!

$\mathrm{Tr}\, F_A \wedge * F_A = \delta^{(4)}$

$$Y = \frac{\mathbb{C} \setminus 0}{\left(z \mapsto z e^{i\theta}\right)} \curvearrowleft U(1)$$

$$V(\varepsilon) = i\varepsilon\left(z\partial_z - \bar{z}\partial_{\bar{z}}\right)$$

$$|dz|^2$$

$$\left(\frac{1}{\varepsilon} + \frac{1}{2i} dz \wedge d\bar{z}\right) e^{\underline{\underline{-\varepsilon z \bar{z}}}} = \Omega$$

$$\Omega|_\infty = 0$$

$$D = d + \iota_{V(\varepsilon)}$$

$$D\Omega = 0$$

$$\int_Y \Omega = \frac{1}{\varepsilon} = \frac{\Omega|_0}{\text{веса действия } U(1) \text{ на } T_0 \bar{Y}}$$

$$\bar{Y} = Y \cup \{0\}$$

полезно <u>(частично)</u> компактифицировать

$\mathcal{M}_k^{\text{framed}} \hookrightarrow \widetilde{\mathcal{M}}_k^{\text{framed}}$ = instantons on noncommutative $\mathbb{R}^4$

$$(dA + A * A)^{+} = 0$$

$*$ - product

$$\frac{\partial}{\partial z_i}$$

$$A = A_\mu dx^\mu \qquad A_\mu(z_1, z_2, \bar{z}_1, \bar{z}_2)$$

$$[z_1, z_2] = 0 \qquad [z_1, \bar{z}_1] + [z_2, \bar{z}_2] = -\zeta$$

$$[\bar{z}_1, \bar{z}_2] = 0$$

$$[c_i, c_j^{+}] = \delta_{ij} \quad i,j = 1,2$$

e.g. $z_i = \sqrt{\zeta/2}\, c_i^{+}$

$\bar{z}_i = \sqrt{\zeta/2}\, c_i$

$$\widetilde{\mathcal{M}}_K^{\text{framed}} = \left\{ (B_1, B_2, I, J) \;\middle|\; \begin{array}{l} I: N \to K \\ J: K \to N \\ B_1, B_2 : K \circlearrowleft \end{array} \right\} / U(K)$$

$$\cong \left\{ (B_1, B_2, I, J) \;\middle|\; \begin{array}{l} \text{Stable } \mathbb{C}[B_1, B_2] I(N) = K \\ \mu_{\mathbb{C}} = 0 \end{array} \right\} / GL(K)$$

$$\mu_{\mathbb{C}} = [B_1, B_2] + IJ = 0$$

$$\mu_{\mathbb{R}} = [B_1, B_1^\dagger] + [B_2, B_2^\dagger] + II^\dagger - J^\dagger J = \zeta \cdot 1$$

$$\zeta > 0$$

$$\widetilde{\mathcal{M}}_K^{\text{framed}} \;\; \big/ GL(K)$$

$$\downarrow \pi$$

$$\overline{\mathcal{M}}_K^{\text{framed}} = \left\{ (B_1, B_2, I, J) \;\middle|\; \mu_{\mathbb{C}} = \mu_{\mathbb{R}} = 0 \right\} / U(K)$$

$$\overline{\mathcal{M}}_K^{\text{framed}} \hookleftarrow \mathcal{M}_K^{\text{framed}} = \left\{ (g B_1 g^{-1}, g B_2 g^{-1}, g I, J g^{-1}) \;\middle|\; \mu_{\mathbb{C}} = \mu_{\mathbb{R}} = 0,\ \in U(K) \right\} \cancel{/U(K)}$$

$$g \in U(K) \qquad id = \mathrm{Stab}(B_1, B_2, I, J)$$

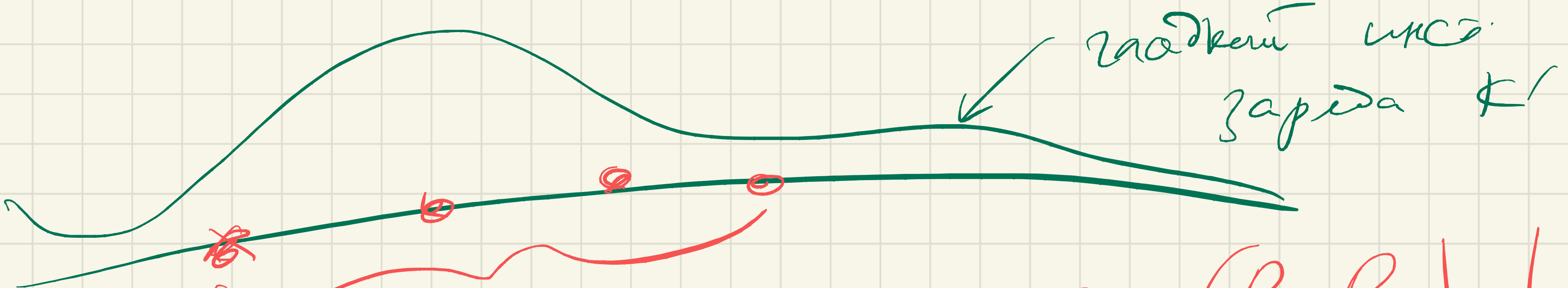
$$\overline{\mathcal{M}} \setminus \mathcal{M} = \{ (B_1, B_2, I, J) \mid K = K' \oplus K'' $$

$$B_1, B_2 = \begin{pmatrix} * & 0 \\ 0 & * \end{pmatrix} \begin{matrix} K' \\ K'' \end{matrix}$$

$I(N) \subset K'$

$J|_{K''} = 0$

$[B_1, B_2]|_{K''} = 0$



K'' точек − спектр $(B_1, B_2)|_{K''}$

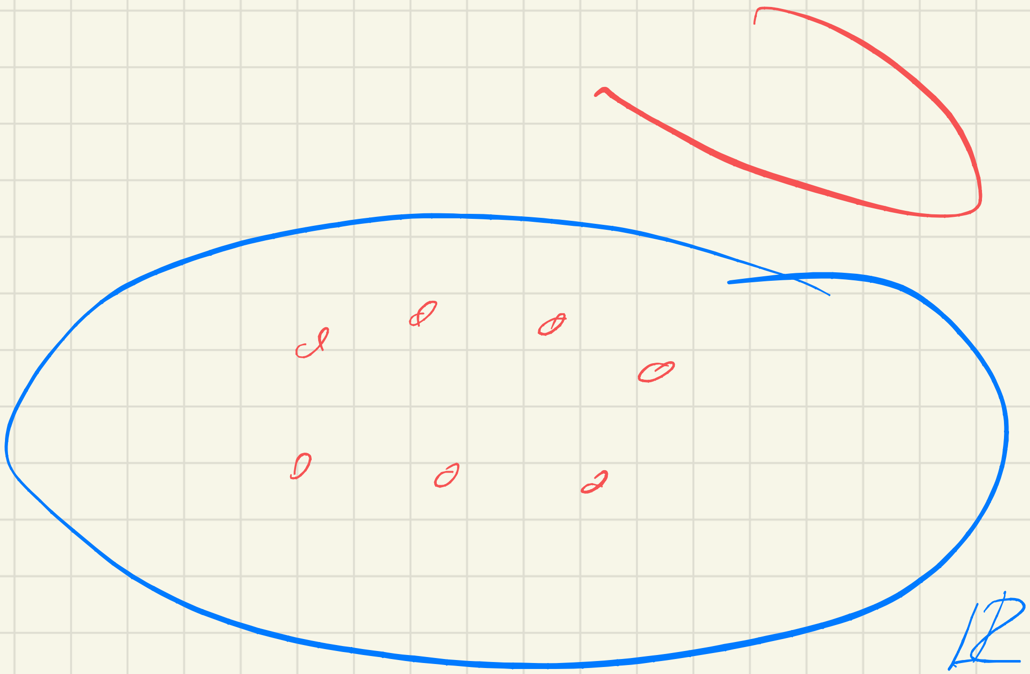
6 $\quad \overline{\mathcal{M}}_k^{\text{framed}}$

locus

$J = 0 \Longrightarrow I = 0$

$(B_1, B_2) = \mathrm{diag}(\beta^a_{1,2})_{a=1}^{k}$

$\mathrm{Sym}^k(\mathbb{R}^4)$



$\mathbb{R}^4$

$\pi \quad \overline{\mathcal{M}}_k^{\text{framed}} \supset \mathrm{Hilb}^{[k_1]}(\mathbb{C}^2) \times \cdots \times \mathrm{Hilb}^{[k_N]}(\mathbb{C}^2)$

$\sum_{i=1}^{N} k_i = k$

$\overline{\mathcal{M}} \supset \mathrm{Sym}^k(\mathbb{R}^4)$

$N = 1$

$\mathcal{N} = \mathbb{C}^N$

$U(N)$

переиде α B_1, B_2, k

$$X = \alpha B_1 + \beta B_2^\dagger$$

$$|\alpha|^2 + |\beta|^2 = 1$$

$$Y = -\bar{\beta} B_2 + \bar{\alpha} B_1^\dagger$$

$$e = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$e = \alpha I + \beta J^\dagger$$

$$e^\dagger = \bar{\alpha} I^\dagger + \bar{\beta} J$$

$N = 1$

$$[X, Y] \cancel{=} -ee^\dagger \cancel{+} \underline{\underline{\varsigma}} \cdot 1$$

— Calogero–Moser phase space

$$\tilde{\mathcal{M}}^{\text{framed}} \subset \tilde{\mathcal{M}}^{\text{adjoint}}$$

$$\tilde{\mathcal{M}}^{\text{adjoint}} = \left\{ (B_1, B_2, B_3, B_4, I, J) \,\middle|\, \begin{array}{l} \mu_{\mathbb{C}} + [B_3, B_4]^{\dagger} = 0 \\ [B_1, B_3] + [B_4, B_2]^{\dagger} = 0 \\ [B_1, B_4] - [B_3, B_2]^{\dagger} = 0 \\ \sum_{a=1}^{4} [B_a, B_a^{\dagger}] + II^{\dagger} - J^{\dagger}J = \zeta \cdot 1 \end{array} \right\} \Big/ U(k)$$

$\zeta > 0$

$\|JB_3\|^2 + \|JB_4\|^2$

$\|B_3 I\|^2 + \|B_4 I\|^2$

$$\boxed{\begin{array}{l} B_3 I + (J B_4)^{\dagger} = 0 \\ B_4 I - (J B_3)^{\dagger} = 0 \end{array}}$$

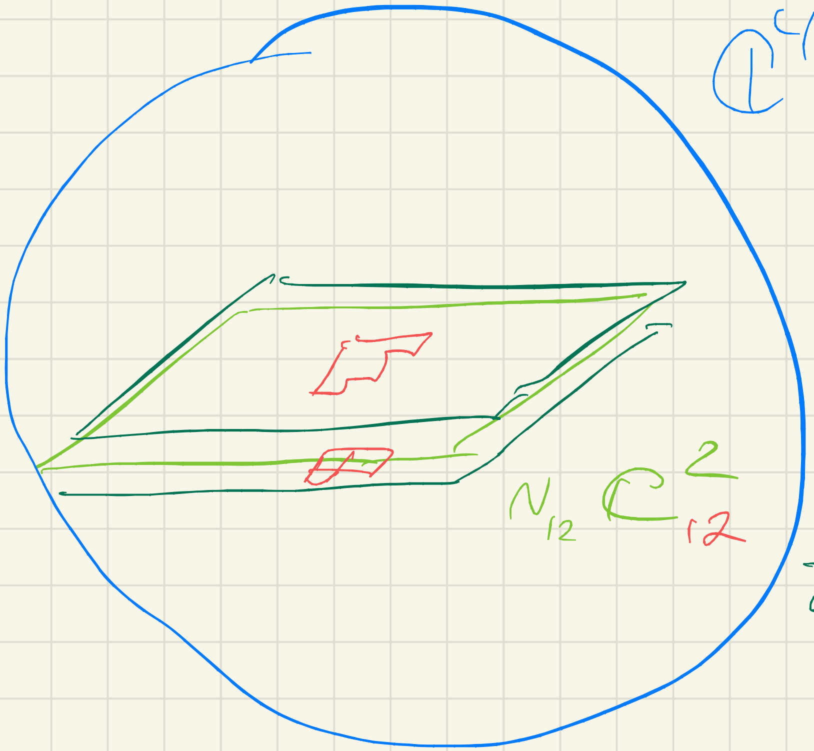
$$\vec{\mathcal{E}} \in \mathbb{R}^7 \otimes \mathrm{Lie}\, U(k)$$

зралнение

$$\| \quad \|^2 + \|\mu_{\mathbb{C}} + [B_3, B_4]^{\dagger}\|^2 + \|[B_1, B_3] + [B_4, B_2]^{\dagger}\|^2 + \|[B_1, B_4] - [B_3, B_2]^{\dagger}\|^2 = \|\mu_{\mathbb{C}}\|^2 + \|[B_3 B_4]\|^2

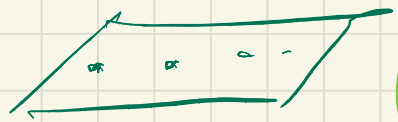
$N = 2^A$

$\mathbb{C}^4_{1234}$



$N_{12} \{$ $\quad N_{12}\, \mathbb{C}^2_{12}$

$\times \; \mathbb{C}$



$\vec{a} = (a_1 \quad a_{N_{12}})$

$SU(4)$

$\cup$

$SU(2)_{12} \times U(1) \times SU(2)_{34}$

$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \quad + \quad \swarrow \searrow \quad = \quad = \begin{pmatrix} z_3 \\ -z_4 \end{pmatrix}$

$\frac{\varepsilon_1 - \varepsilon_2}{2}$ $\quad \frac{\varepsilon_3 - \varepsilon_4}{2}$

$\frac{\varepsilon_1 + \varepsilon_2}{2} = -\frac{\varepsilon_3 + \varepsilon_4}{2}$

случайная переменная

$\vec{\lambda} = \left(\lambda^{(12,\alpha)} \quad \alpha = 1, \dots, N_{12} \right)$ $\quad q$

$\mu^{(12)}_{\vec{\lambda}} (\vec{a}, \vec{\varepsilon})$

$Z_{12} = \sum_{\vec{\lambda}} \mu^{(12)}_{\vec{\lambda}} \, q^{(\vec{\lambda})} \Big| \; \vec{\varepsilon} = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4)$

$\sum_a \varepsilon_a = 0$

Background

$$S \subset \mathbb{C}^4 \times \mathbb{C}$$

$$\left\{ \left(z_3 = z_4 = 0 ,\ z_5 = a_\alpha \right) \;\middle|\; \alpha = 1, \dots, N \right\}$$

Z_S

$a : \binom{4}{2} \to$ divisor on $\mathbb{C}$ of multiplicity $N^{(\alpha)}$

$N : \binom{4}{2} \to \mathbb{Z}_{\geq 0}$

SPIKED INSTANTONS

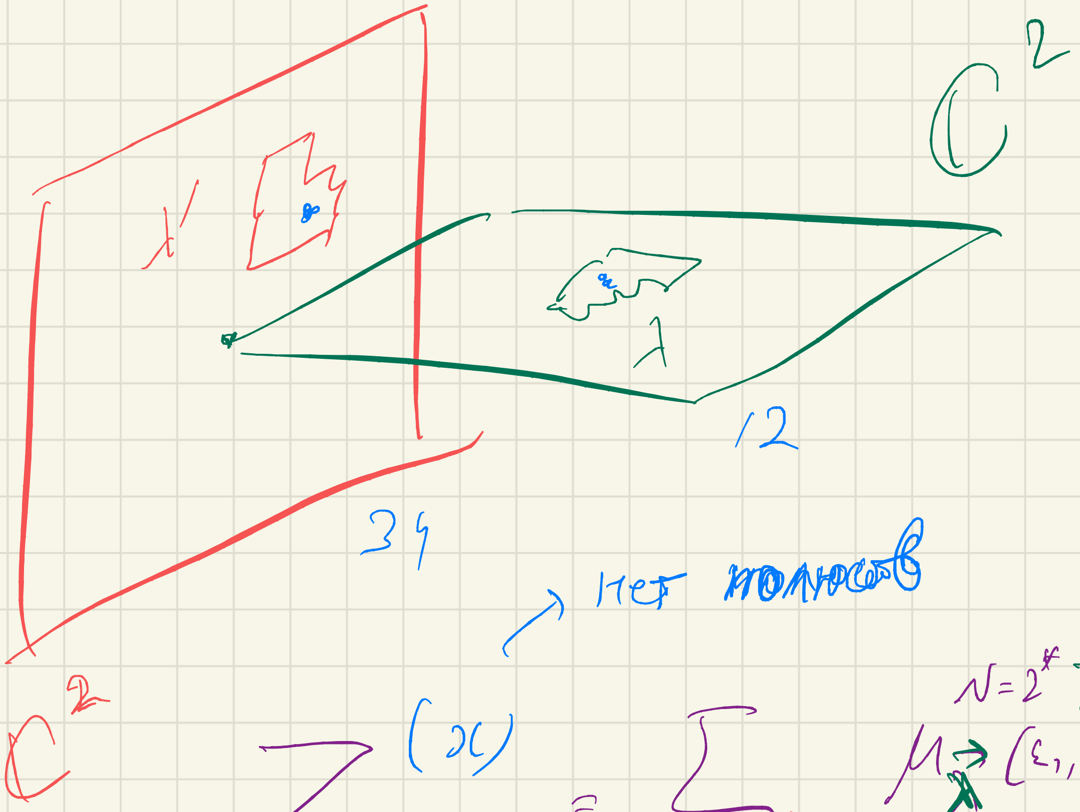
$$S_N = \left\{ \vec{z} \in \mathbb{C}^5 \;\middle|\; \exists\ \alpha, \beta,\ 1 \leq \alpha < \beta \leq 4 \right.$$

$$\left. z_\alpha = z_\beta = 0 ,\ z_5 = a_k^{(\alpha\beta)} \quad k = 1, \dots, N_{\alpha\beta} \right\}$$

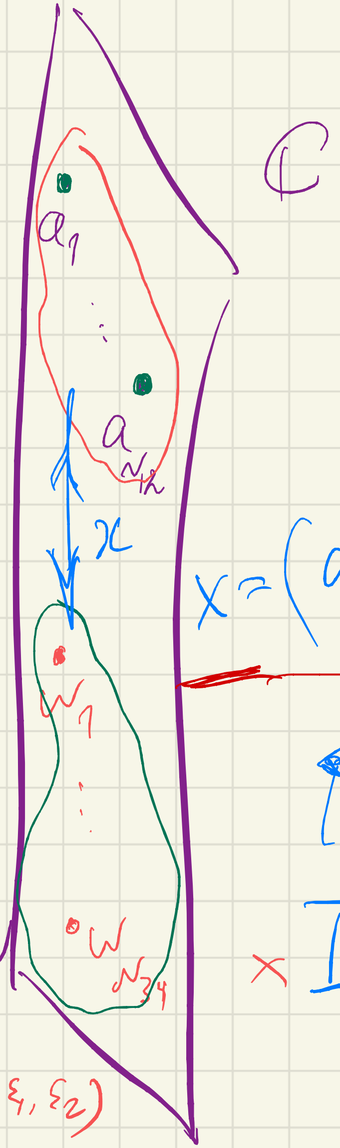
6

CROSSED INSTANTONS

$N_{12} \neq 0, \quad N_{34} \neq 0$



$\to$ нет полюсов



$(i,j) \in \lambda^{(12,\alpha)}$

$(i',j') \in \lambda'^{(34,\beta)}$

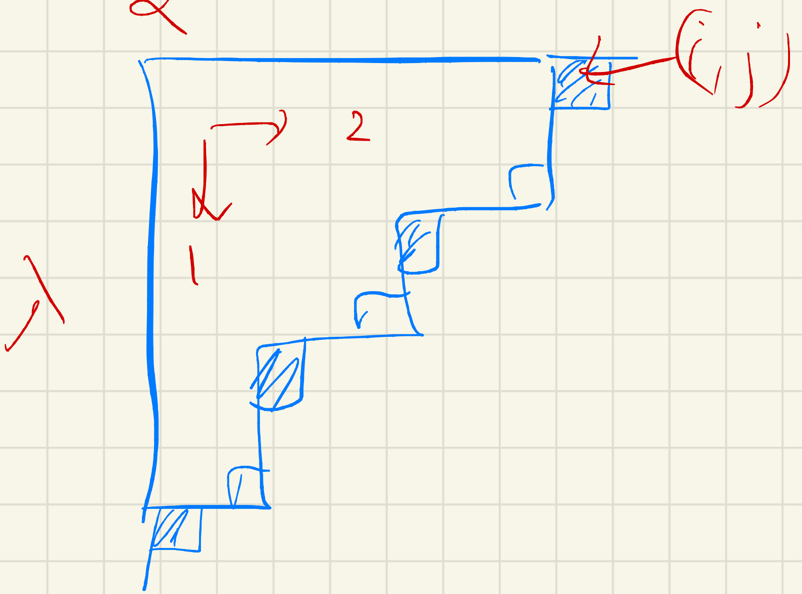
$$x = \big(a_\alpha - w_\beta + \varepsilon_1(i-1) + \varepsilon_2(j-1) - \varepsilon_3(j'-1) - \varepsilon_4(j'-1)\big)$$

$$Z^{(x)}_{N_{12},N_{34}} = \sum_{\vec\lambda,\vec\lambda'} \mu_{\vec\lambda}(\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4)\, \mu_{\vec\lambda'}(\varepsilon_3,\varepsilon_4,\varepsilon_1,\varepsilon_2) \times I_{\vec\lambda,\vec\lambda'}(x)$$

$N = 2^{\#}$ $\vec a$,

$N = 2^{\#}$ $\vec w$,

$$I_{\vec{\lambda},\vec{\lambda}'} \sim \frac{\displaystyle\prod_{\blacksquare\in\partial_+\lambda,\ \blacksquare'\in\partial_+\lambda'} \left(x_{\blacksquare,\blacksquare'}\right) \times \prod_{\square\in\partial_-\lambda,\ \square'\in\partial_-\lambda'} \left(x_{\square,\square'}+\varepsilon_1+\varepsilon_2\right)^{?}}{\displaystyle\prod_{\blacksquare\in\partial_+\lambda,\ \square\in\partial_-\lambda'} \left(x_{\blacksquare,\square}\right) \prod_{\square\in\partial_-\lambda,\ \blacksquare\in\partial_+\lambda'} \left(x_{\square,\blacksquare}\right)}$$

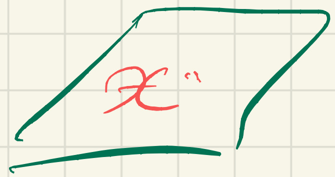


$$x_{\blacksquare,\blacksquare'} = a_\alpha + \varepsilon_1(i-1) + \varepsilon_2(j-1) - w_\beta - \varepsilon_3(i'-1) - \varepsilon_4(j'-1)$$

$$Z_{N_{12,34}}$$ не имеет полюсов по переменной $x = \frac{1}{N_{12}} \sum_\alpha a_\alpha - \frac{1}{N_{34}} \sum_\beta w_\beta$

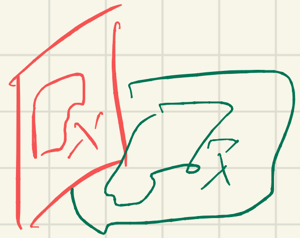
$$\| $$

$$\left\langle \mathcal{X}^{(12)}_{q}(\vec{w}) \right\rangle^{\vec{a},q}_{C^2_{12}} = \left\langle \mathcal{X}^{(34)}_{q}(\vec{a}) \right\rangle^{\vec{u},q}_{C^2_{34}}$$



qq – характер = наблюдаемая $\mathcal{X}$

$$\mathcal{X}^{(12)}(\vec{w})\Big|_{\vec{\lambda}} = \sum_{\vec{\lambda}'} \mu_{\vec{\lambda}'} I_{\vec{\lambda},\vec{\lambda}'}(x)$$

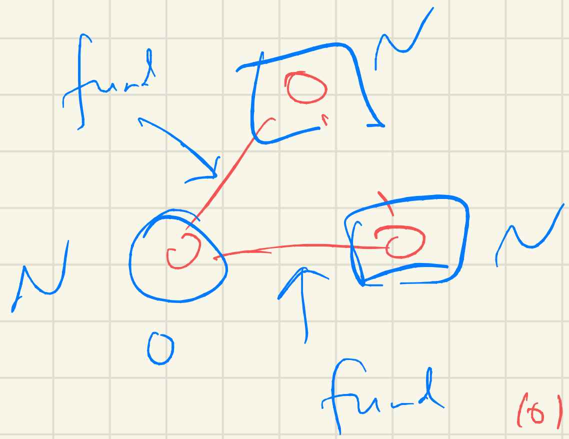
$$\mathcal{X}_{\vec{\lambda}'} = \sum_{\vec{\lambda}'}$$ 

В частности $\underline{A_1 \text{ теория}}$ ($U(N) + 2N$ fund hyper)

$\mathbb{Z}_3$ orbifold of $\mathcal{N}=2^*$

$(z_3, z_4) \longmapsto (\omega z_3, \omega^{-1} z_4)$ $\qquad \omega^3 = 1$

$q \longmapsto (q_0, q_1, q_2)$



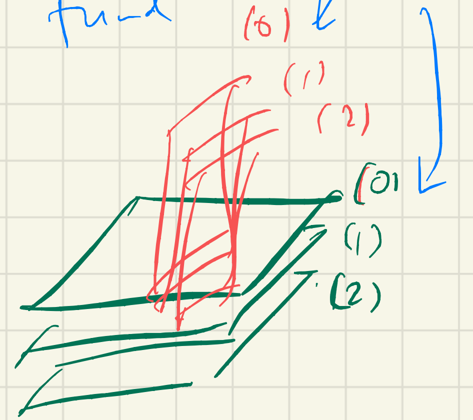
$q_1 = 0$

$q_2 = 0$

$N_{34} \Longmapsto N_{34}^{(0)}, N_{34}^{(1)}, N_{34}^{(2)}$

$N_{12} \longmapsto (\underset{\substack{\shortparallel \\ N}}{N_{12}^{(0)}}, \underset{\substack{\shortparallel \\ N}}{N_{12}^{(1)}}, \underset{\substack{\shortparallel \\ N}}{N_{12}^{(2)}})$

$N_{34}^{(1)} = 1$



$$q_0 = q \qquad q_1 = q_2 = 0$$

$$\prod_{w=0}^{2} \left(\prod_{\alpha=1}^{N_{12}^{(w)}} \prod_{(i,j)\in\lambda^{(12;w;\alpha)}} q_w \times \prod_{\beta=1}^{N_{34}^{(w)}} \prod_{(i,j)\in\lambda^{(34;w;\beta)}} q_{(w+i-j) \bmod 3} \right)$$

$$\underset{q_w}{\not\!P}$$

$\neq 0 \Rightarrow \lambda^{(12;w;\alpha)} = \emptyset$ $w=1,2$, $\forall\alpha$, $\lambda^{(34;w;\beta)} = \emptyset$ $w=1,2, \forall\beta$

$\lambda^{(34;0;\beta)} = \emptyset$, or $\square$

Простейший qq - характер

A_1 $\qquad N_{34} = \left(N_{34}^{(0)} = 1,\ N_{34}^{(1)} = N_{34}^{(2)} = 0\right)$

$$\mathcal{H} = Y(x+\varepsilon_1+\varepsilon_2) + \mathfrak{q}\,\frac{P(x)}{Y(x)}$$

$P(x) = \prod_{f=1}^{w} (x - m_f)$

$$\mathcal{H}\big|_{\vec{\lambda}} = Y\big|_{\vec{\lambda}}(x+\varepsilon_1+\varepsilon_2) + \mathfrak{q}\,\frac{P(x)}{Y_{\vec{\lambda}}(x)}$$

$$Y_{\vec{\lambda}}(x) = \prod_{\alpha=1}^{N_{12}} \frac{\prod_{\square\in\partial_{+}\lambda^{(\alpha)}}(x - a_\alpha - c_\square)}{\prod_{\square\in\partial_{-}\lambda^{(\alpha)}}(x - a_\alpha - c_\square - \varepsilon_1 - \varepsilon_2)}$$

$$M_{adj} = \mathcal{E}_3$$

$$\mathcal{M}^{adj}_{K} = \{ (B_1, B_2, B_3, B_4, I, J) \} \Big/ \text{Equations} \Big/ U(k)$$

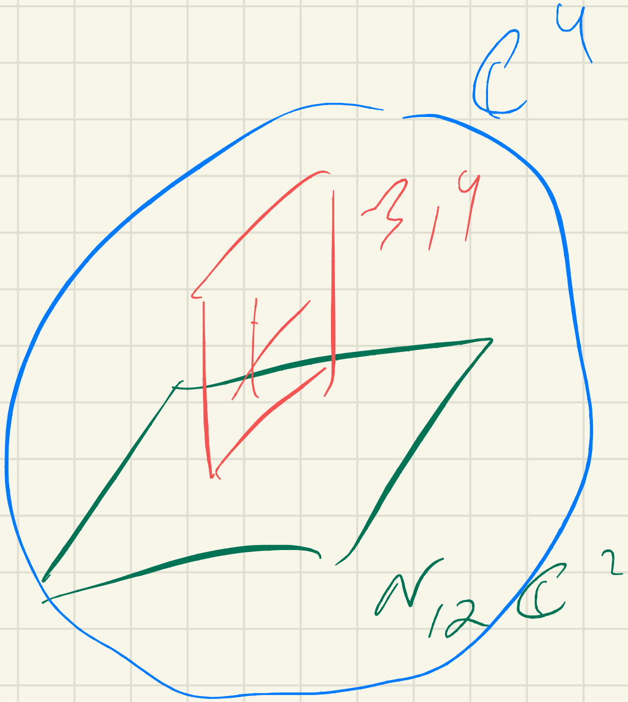
$$\mathcal{M}^{spiked} = \{ (B_a, I_A, J_A) \mid \begin{array}{l} a = 1, \dots, 4 \\ A = 12, 13, 14, 23, 24, 34 \end{array} \}$$

$$Z^{\text{inst}}_{S} = \sum_{k} q^{k} \text{``}\int_{\mathcal{M}_{S}^{\text{spiked}}} 1\text{''}$$

$$\text{``} = \int \overbrace{D\cancel{X}\, D\Psi}^{\text{var}}\; \overbrace{DX\, DH}^{\text{eqn}}\; \overbrace{D\sigma\, D\bar{\sigma}\, D\eta}^{\text{Lie}\, U(k)} \Big/ U(k)$$

$$\underset{b}{X}\quad \underset{f}{\Psi}\qquad \underset{f}{X}\quad \underset{b}{H}\qquad \underset{B}{\sigma}\quad \underset{B}{\bar{\sigma}}\quad \underset{F}{\eta}$$

$$Z^{\text{inst}, \, N=2^*}_{N_{12}} = \sum_{k} q^k \int_{\mathcal{M}_k^{\text{adj}}} 1 = \sum_{k} q^k \int_{\mathcal{M}_k^{\text{ADHM}}} C\left(\epsilon_3, T\mathcal{M}_k^{\text{ADHM}}\right)$$



$\epsilon_3 \to \infty$

$q \to 0$

$\longrightarrow$ pure $N=2$

ИНВАРИАНТ

broke $3 \leftrightarrow 4$ symm.

$$T^{adj} = P_3 NK^* + P_3 q_{1,2} KN^* - P_{123} KK^*$$

Real

$$T^{adj} = T^{adj} + T^{adj\,*} = P_{34}\left(NK^* + q_{1,2} KN^*\right) - P_{1234} KK^*$$

$$q_{abc} = q_a q_b q_c$$

$$P_a = 1 - q_a$$

$$P_{ab} = P_a P_b$$

$$q_{1234} = 1$$

$$P_a^* = 1 - q_a^{-1} = -P_a q_a^{-1}$$

$$e^{\beta m} = q_3$$

$$\mu^{\mathrm{ad}=2^+} = \prod_{x \in T} \frac{x + \varepsilon_3}{x} = \prod_{Y \in T^+} \frac{(Y+\varepsilon_3)(-Y-\varepsilon_4)}{Y(-Y+\varepsilon_1+\varepsilon_2)}$$