Non-diagonal problem Hamiltonian for adiabatic quantum computation

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Overview

Adiabatic quantum computation

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adiabatic condition: $T_N \sim 1/\Delta_N^2$

Appeals of AQC

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- multiple interrelations with condensed matter physics

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- there is strong evidence that T can often scale unfavorably with the problem size N

Monotone not-all-equal 3-satisfiability (MNAE3SAT)

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MNAE3SAT is NP-complete

MNAE3SAT as a binary optimization problem

MNAE3SAT = binary optimization problem with the cost function

$$H_{\mathrm{p}}^{\mathrm{cl}}(z) = \sum_{(i,j,m)\in\mathcal{C}} C_{ijm}^{\mathrm{cl}}(z)$$

with

$$C_{ijm}^{\mathrm{cl}}(z) = \begin{cases} 1 & \text{if } \; z_i = z_j = z_k, \\ 0 & \text{otherwise.} \end{cases}$$

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z is a satisfying assignment $\quad \Leftrightarrow \quad H_{\rm p}^{\rm cl}(z) = 0$

Conventional H_{p} for MNAE3SAT

$$H_{\mathrm{p}} = \sum_{(i,j,m)\in\mathcal{C}} C_{ijm}$$

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H_p is frustration-free:

$$H|z\rangle = 0 \quad \Leftrightarrow \quad \forall\ (i,j,m) \in \mathcal{C} \quad C_{ijm}|z\rangle = 0.$$

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z is a satisfying assignment $\Leftrightarrow$ z is a gs, i.e. $H_\mathrm{p}|z\rangle = 0$

Bottlenecks of AQC

Bottleneck of AQC = avoided level crossings with $\Delta \sim e^{-N^\alpha}$

Two types of bottlenecks

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- Quantum phase transitions

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- Many-body localised (glassy) phase [Altshuler, Krovi, Roland, 2010; Laumann *et al.* 2015; Knysh 2016; ...]

“Conventional” AQC

$$H_t = (1 - \frac{t}{T}) H_0 + \frac{t}{T} H_{\mathrm{p}}$$

$$H_{\mathrm{p}} = \sum_{i,j} J_{ij} \sigma_i^z \sigma_j^z + \sum_i h_i \sigma_i^z$$

$$\hat{H}_0 = \sum_i \sigma_i^x$$

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- **modify** H_{p}

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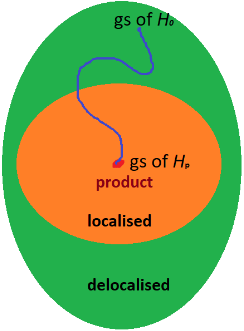

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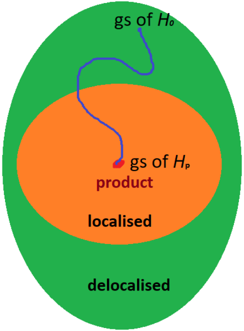

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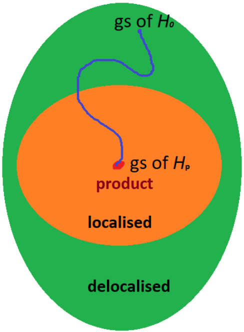

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- product states are ultimately localised

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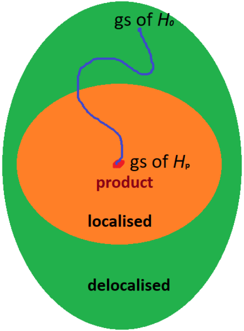

- in disordered systems eigenstates can be many-body localised (MBL)
- MBL entails small energy gaps
- product states are ultimately localised
- eigenstates of H_p are of product form, hence the evolution inevitably traverses MBL phase

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- ground state of $H_{\rm p}$ is of product form for a purpose – it should be easily measurable
- however, excited states of $H_{\rm p}$ are also product states – absolutely unnecessary for computation!
- the idea is to introduce $H_{\rm p}^{\rm ent}$ with a product ground state and entangled excited states

Non-diagonal problem Hamiltonian

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Reminder:

$$H_{\rm p} = \sum_{(i,j,m)\in\mathcal{C}} C_{ijm}$$

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A problem Hamiltonian (generically) non-diagonal in comp. basis:

$$H_{\mathrm{p}}^{\mathrm{ent}} = \sum_{(i,j,m)\in\mathcal{C}} C_{ijm} A_{ijm} C_{ijm}$$

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A_{ijm} not necessarily acts on spins i, j, m

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A specific choice of A_{ijm}:

$$A_{ijm} = 2 + \sigma_i^x \sigma_j^x \sigma_m^x + \sigma_r^x \sigma_s^x$$

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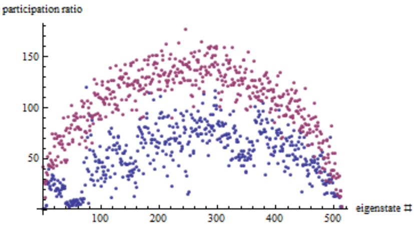
Locality issue: $H_{\mathrm{p}}^{\mathrm{ent}}$ is 4-local (while H_{p} is only 2-local)

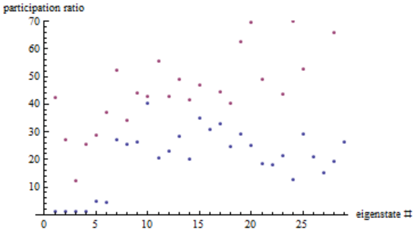
Entanglement of eigenstates of $H_{\mathrm{p}}^{\mathrm{ent}}$

Participation ratio – figure of merit for entanglement:

$$R(\Psi) = \left(\sum_{\mu=1}^{2^N} |\Psi_\mu|^4 \right)^{-1} .$$

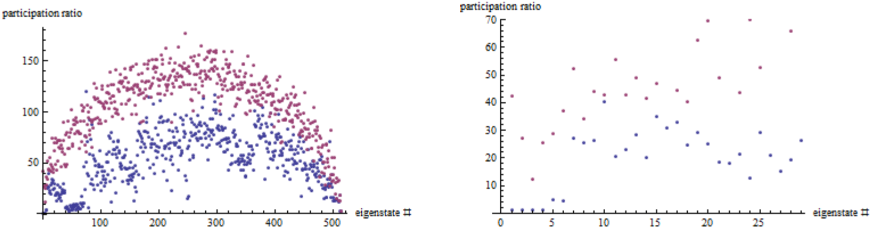
Entanglement of eigenstates of $H_{\mathrm{p}}^{\mathrm{ent}}$





Participation ratios of eigenstates of $H_{\mathrm{p}}^{\mathrm{ent}}$ (blue dots) compared to those of eigenstates of a nonintegrable Ising model.

Entanglement of eigenstates of $H_{\mathrm{p}}^{\mathrm{ent}}$



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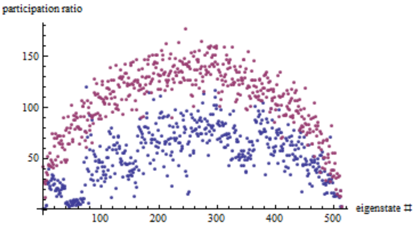

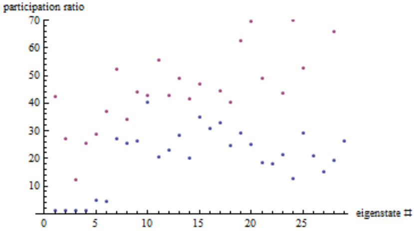

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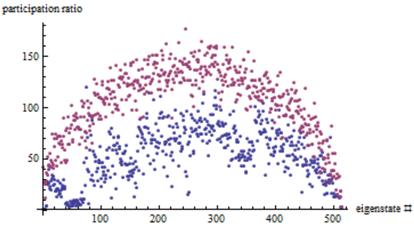

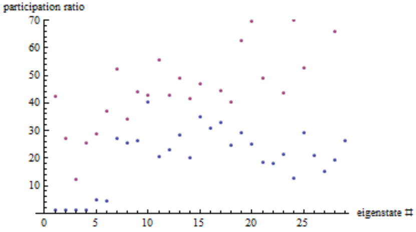

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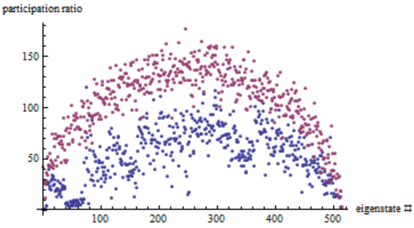
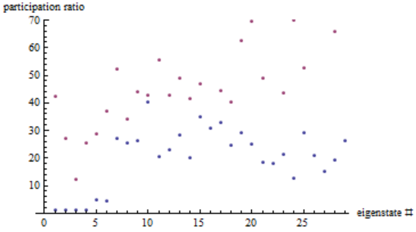
Entanglement of eigenstates of $H_{\rm p}^{\rm ent}$





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- work in progress...

Non-diagonal problem Hamiltonian - generalisation

Diagonal frustration-free problem Hamiltonian:

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Product ground state $|z\rangle$ with zero energy: $\quad H_{\rm p}|z\rangle = 0$

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$$H_{\rm p}^{\rm ent} = \sum_{\substack{(i,j,m)\in\mathcal{C}\\(n,l,q)\in\mathcal{C}}} C_{nlq} A_{ijm}^{nlq} C_{ijm}, \qquad A_{ijm}^{nlq} > 0$$

has the same ground state, $\quad H_{\rm p}^{\rm ent}|z\rangle = 0$
but (generically) entangled excited eigenstates.

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- a problem Hamiltonian with entangled excited states can always be chosen
- entanglement comes for the price of increased non-locality
- such problem Hamiltonians may help in evading localisation bottlenecks of AQC
- more work is needed to evaluate their performance

arXiv:1811.09453

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Thank you for your attention!