Course Syllabus

Skoltech

Course Title	Integrable Many-Body Systems and Nonlinear Equations
Course Title (in Russian)	Интегрируемые многочастичные системы и нелинейные уравнения
Lead Instructor	Zabrodin, Anton

1. Annotation

Course Description

This course is devoted to many-body integrable systems of classical mechanics such as Calogero-Moser, Ruijsenaars-Schneider and their spin generalizations. These systems play a significant role in modern mathematical physics. They are interesting and meaningful from both mathematical and physical points of view and have important applications and deep connections with different problems in mathematics and physics. The history of integrable many-body systems starts in 1971 from the famous Calogero-Moser model which exists in rational, trigonometric or hyperbolic and (most general) elliptic versions. Later it was discovered that there exists a one-parametric deformation of the Calogero-Moser system preserving integrability, often referred to as relativistic extension. This model is now called the Ruijsenaars-Schneider system. In its most general version the interaction between particles is described by elliptic functions.

The integrable many-body systems of Calogero-Moser type have intimate connection with nonlinear integrable equations such as Korteveg-de Vries and Kadomtsev-Petviashvili (KP) equations. Namely, they describe dynamics of poles of singular solutions (in general, elliptic solutions) to the nonlinear integrable partial differential equations. The Ruijsenaars-Schneider system plays the same role for singular solutions to the Toda lattice equation.

In this course the algebraic structure of the integrable many-body systems will be presented. The Lax representation and interrals of motion will be obtained using the corerspondence with the equations of the KP type. The necessary material about the latter will be given in the course. The construction of the spectral curves will be discussed.

Course Description (in Russian)

Этот курс посвящен интегрируемым многочастичным системам классической механики таким как системы Калоджеро-Мозера, Руйсенарса-Шнайдера и их спиновые обобщения. Эти системы играют значительную роль в современной математической физике. Они интересны и содержательны как с математической, так и с физической точек зрения и имеют важные приложения и глубокие связи с различными проблемами в математике и физике. История интегрируемых систем многих частиц начинается в 1971 году со знаменитой модели Калоджеро-Мозера, которая существует в рациональной, тригонометрической или гиперболической и (наиболее общей) эллиптической версиях. Позднее было обнаружено, что существует однопараметрическая деформация системы Калоджеро-Мозера, сохраняющая интегрируемость, которая известна также как ее релятивистское обобщение. В настоящее время эта модель называется системой Руйсенарса-Шнайдера. В ее наиболее общей версии взаимодействие между частицами описывается эллиптическими функциями.

Интегрируемые системы типа Калоджеро-Мозера имеют тесную связь с нелинейными интегрируемыми уравнениями такими как уравнение Кортевега-де Фриза и уравнение Кадомцева-

Петвиашвили (КП). А именно, они описывают динамику полюсов сингулярных решений (в общем случае эллиптических) нелинейных интегрируемых уравнений в частных производных. Система Руйсенарса-Шнайдера играет ту же роль для сингулярных решений уравнения решетки Тоды.

В данном курсе будет представлена алгебраическая структура интегрируемых многочастичных систем. С помощью соответствия с уравнениями типа КП будет получено представление Лакса и найдены интегралы движения. Необходимый материал о нелинейных интегрируемых уравнениях будет дан в курсе. Будет также обсуждаться конструкция спектральных кривых.

2. Basic Information

Course Academic Level MSc PhD

Number of ECTS credits 6

Course Prerequisites / Recommendations

Students should be familiar with linear algebra and complex analysis.

Type of Assessment Graded

Mapping from grades to percentage:

A: 86

B: 76

C: 66

D: 56

E: 46

F: 0

Term Term 3-4

Students of Which Programs do You Recommend to Consider this Course as an Elective?

BSc Programs	Masters Programs	PhD Programs
		Mathematics and Mechanics

Maximum Number of Students

	Maximum Number of Students
Overall:	10
Per Group (for seminars and labs):	10

Course Stream Entrepreneurship and Innovation (E&I)

3. Course Content

Lecture, lab and seminar hour distribution among topics

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Calogero-Moser system	Introduction to Calogero-Moser systems (rational, trigonometric and elliptic)	2	0	0
Ruijsenaars-Schneider system	Introduction to the Ruijsenaars-Schneider systems (relativistic extension of Calogero-Moser)	2	0	0
Spin genaralization of systems of Calogero-Moser type	Spin generalization of systems of Calogero-Moser type: integrals of motion, spectral curves	4	0	0
Equations of KP type	Nonlinear integrable equations of KP and Toda type: Lax representation, auxiliary linear problems	4	0	0
Elliptic solutions to the KP equation	Derivation of dynamics of poles of elliptic solutions to the KP equation	2	0	0
Elliptic solutions to the Toda equation	Derivation of pole dynamics of elliptic solutions to the Toda equation	2	0	0
Hamiltonian structure	Hamiltonian structure of integrable many-body systems	2	0	0
Matrix KP equation	Dynamics of poles of elliptic solutions to the matrix KP equation	4	2	0

4. Learning Outcomes

Please choose framework for learning outcomes Knowledge-Skill-Experience

Knowledge
Students will understand the algebraic structure of integrable many-body systems

Skill
Students will be able to derive the dynamics of poles of singular solutions to nonlinear partial differential equations

Experience
The experience of algebraic analysis of integrable systems

Learning outcomes of the course are specified in the steering document of the educational program in accordance with the Federal State Educational Standard on the website in Education section / Результаты освоения по учебному элементу в соответствие с ФГОС отражены в описании образовательной программы в разделе Образование

5. Assignments and Grading

Assignment Types

Assignment Type	Assignment Summary	% of Final Course Grade
Exercise	Problems for home work on each topic	50
Final Exam	Exam in the end of the course	50

6. Assessment Criteria

Select Assignment 1 Type Problem Set

Input or Upload Sample of Assigment 1:

Input Sample of Assignment 1

To obtain equations of motion for the Calogero-Moser model from the Lax equation.

Assessment Criteria for Assignment 1

The complete and correct derivation.

Select Assignment 2 Type Homework Assignments

Input or Upload Sample of Assigment 2:

Input Sample of Assignment 2

To obtain the Lax representation for the Ruijsenaars-Schneider system

Complete and correct derivation

Input or Upload Sample of Assigment 3:

Input or Upload Sample of Assigment 4:

Input or Upload Sample of Assigment 5:

Input or Upload Sample of Assigment 6:

Input or Upload Sample of Assigment 7:

Input or Upload Sample of Assigment 8:

Input or Upload Sample of Assigment 9:

In the next question we ask you to define general categories of the course. What does your course teaches in broad terms?

7. Textbooks and Internet Resources

You can request at most two required textbooks. Additionaly, you can suggest up to nine recommended textbooks.

Required Textbooks	ISBN-13 (or ISBN-10)
А.М. Переломов, Интегрируемые системы классической механики и алгебры Ли, Москва, Наука, 1990	5-02-013826-6

Papers	DOI or URL
A.Zabrodin, Elliptic solutions to integrable nonlinear equations and many-body systems,Journal of Geometry and Physics 146 (2019) 103506	10.1016/j.geomphys.2019.103506

8. Facilities

Software
None

9. Additional Notes