Course Syllabus

Skoltech

Course Title	Modern Dynamical Systems
Course Title (in Russian)	Современные динамические системы
Lead Instructor	Skripchenko, Alexandra
Co-Instructor	Lando, Sergey

1. Annotation

Course Description

Dynamical systems in our course will be presented mainly not as an independent branch of mathematics but as a very powerful tool that can be applied in geometry, topology, probability, analysis, number theory and physics. We consciously decided to sacrifice some classical chapters of ergodic theory and to introduce the most important dynamical notions and ideas in the geometric and topological context already intuitively familiar to our audience. As a compensation, we will show applications of dynamics to important problems in other mathematical disciplines. We hope to arrive at the end of the course to the most recent advances in dynamics and geometry and to present (at least informally) some of results of A. Avila, A. Eskin, M. Kontsevich, M. Mirzakhani, G. Margulis.
In accordance with this strategy, the course comprises several blocks closely related to each other. The first three of them (including very short introduction) are mainly mandatory. The decision, which of the topics listed below these three blocks would depend on the background and interests of the audience.

Course Description (in Russian)

Курс о современных динамических системах - это обзор результатов, полученных в нескольких направлениях за последние тридцать лет, причем основной упор делается на задачах, которые пришли из соседних разделов математики - маломерной топологии, теории чисел, геометрии - и оказались решены с помощью методов теории динамических систем или, наоборот, возникли исторически в динамике, но потребовали для своего решения привлечения инструментов из других областей - комбинаторики, теории вероятностей и других. Мы планируем начать с простых и понятных примеров и дойти до знакомства с достаточно свежими работами ряда выдающихся математиков (А. Авилы, А. Эскина, М. Мирзахани, К. МакМаллена, Г. Маргулиса и других).

2. Basic Information

Course Academic Level

Master-level course suitable for PhD students

Number of ECTS credits

6

Course Prerequisites / Recommendations

We expect our audience to be familiar with basic differential geometry, basic topology and basic measure theory.

Type of Assessment

Graded

Mapping from grades to percentage:

A: 86

B: 76

C: 66

D: 56

E: 46

F: 0

Term

Multi-term 3-4

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Data Science Mathematical and Theoretical Physics	Mathematics and Mechanics Physics

Maximum Number of Students

	Maximum Number of Students
Overall:	50
Per Group (for seminars and labs):	30

Course Stream

Science, Technology and Engineering (STE)

3. Course Content

Lecture, lab and seminar hour distribution among topics

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Introduction	We will introduce dynamical systems using the most elementary examples — rotation of the circle and continued fractions.	1	2	
Dynamics and geometry	In this part we will check how dynamical methods can be used to study one of the most classical notions of differ- ential geometry — geodesics on surfaces of negative curvature. Here is our approximate plan: (1) Introduction to hyperbolic geometry. Möbius transformations. Fuchsian groups. (2)The eight geometries of the geometrization conjecture by W. Thurston. (3) Geodesics on surfaces of negative curvature. The geodesic flow and its properties. (4) Geodesic flow on modular curve as a continued fraction map. (5) Teichmüller space. Teichmüller geodesic flow. (6)Counting of simple closed geodesics: results by M. Mirzhakhani.	4	16	

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Dynamics, ergodic theory and topology	In this part we stay again on a Riemann surface but now we would like to have an almost flat metrics on it and to consider related geodesic flow (equivalently, we study measured foliations on such a surface). The purpose of this block is to make a crash course in ergodic theory with a topological interpretation of the main notions and results. The plan is as follows: (1) Interval exchange transformations (IET) as natural generalizations of continued fractions. (2) IET as the first return maps on transversal for measured foliations on oriented surface. Poincaré recurrence theorem. (3) Key ergodic properties: minimality, ergodicity, number of invariant measures (illustrated by IET). (4) Multiplicative ergodic theorem. Topological interpretation of Lya- punov exponents. Sums of Lyapunov exponents as uniform bounds for degrees of holomorphic subbundles.	4	16	

	(5) Anosov and Pseudoanosov diffeomorphisms of surfaces. Introduction to hyperbolic dynamics (Markov partitions, invariant measures etc).			
Dynamics and number theory	This block is dedicated to homogeneous dynamics and its applications to famous conjectures in number theory, such as Oppenheim conjecture (solved) and Littlewood conjecture (still open). We mainly will follow G. Margulis work in this direction.	2	8	
Dynamics and analysis (optional)	This block can be more interesting for fu- ture specialists in mathematical physics and numerical analysis. We plan to discuss the notion of transfer operator and its spectral gap, Perron-Frobenius theorem and its generalization by D. Ruelle, zeta-function and its interpre- tation in terms of transfer operator.	1	4	
Dynamics and probability (optional)	Dynamics and probability. This block is dedicated to thermodynamical formalism and its application for dynamical systems. We discuss beautiful ideas by R. Dobrushin, on the one hand, and R. Bowen — on the other, and show how to find Gibbs	1	2	

	measures for dynamical systems that can be presented as a symbolic shift.			

4. Learning Outcomes

Please choose framework for learning outcomes Knowledge-Skill-Experience

Knowledge
The students will have a relatively deep understanding of the basic ideas and modern problems in dynamical systems

Skill
The students will be able to understand quite advanced papers containing fresh results in dynamical systems

Experience
The students will get experience of a real research work in a fast developing area of modern mathematics

5. Assignments and Grading

Assignment Types

Assignment Type	Assignment Summary	% of Final Course Grade
Problem Set	We provide lists of problems dedicated to every topic (as a homework assignment)	20
Projects	The students give talks based on the modern research papers	70
Final Exam	Those who did not contributed to the home assignments and/or did not give a talk can pass the final exam	10

6. Assessment Criteria

Select Assignment 1 Type Problem Set

Input or Upload Sample of Assigment 1:

Input Sample of Assignment 1

An example of the problem list can be found in the uploaded file.

Or Upload Sample of Assignment 1

Assessment Criteria for Assignment 1

Each problem has a weight, the final grade depends on a the sum of the weights

Select Assignment 2 Type Projects

Input or Upload Sample of Assigment 2:

Input Sample of Assignment 2

A research paper to study is given to each participant, an example is uploaded.

Or Upload Sample of Assignment 2

In case of a successful presentation of a given result in front of the student auditory the student gets a full mark.
Otherwise we evaluate his level of understanding and the efforts used to achieve it.

Select Assignment 3 Type Final Exam

Input or Upload Sample of Assigment 3:

Input Sample of Assignment 3

Each problem costs 3 points. The final mark is given by the minimum of
10 and the sum of all scores. The exam will last 1 hour 20 minutes.

Or Upload Sample of Assignment 3

Input or Upload Sample of Assigment 4:

Input or Upload Sample of Assigment 5:

Input or Upload Sample of Assigment 6:

Input or Upload Sample of Assigment 7:

Input or Upload Sample of Assigment 8:

Input or Upload Sample of Assigment 9:

In the next question we ask you to define general categories of the course. What does your course teaches in broad terms?

7. Textbooks and Internet Resources

You can request at most two required textbooks. Additionaly, you can suggest up to nine recommended textbooks.

Required Textbooks	ISBN-13 (or ISBN-10)
S. Katok, Fuchsian groups, University of Chicago Press, Chicago and London, 1992 (Russian translation: Faktorial Press, Moscow, 2002)	9780226425825
Ya. Sinai, Introduction to ergodic theory Princeton University Press, 1977	9780691081823

Recommended Textbooks	ISBN-13 (or ISBN-10)
W. Thurston, Geometry and topology of three-manifolds, Princeton University Press, 1997 (Russian translation: MCCME, 2001	9780691083049

8. Facilities

Labs for Education

NRU HSE-Skoltech International Laboratory of Representation Theory and Mathematical Physics

9. Additional Notes

Comments

The course takes part on Usacheva, 6 (Math faculty at HSE University) but will be broadcasted online