Skoltech

Course Syllabus

Course Title	Around the Ising Model in 20 Hours
Course Title (in Russian)	Вокруг модели Изинга за 20 часов
Lead Instructor	Shlosman, Semen

1. Annotation

Course Description

This is a course on rigorous results in statistical mechanics, random fields and percolation theory. Some of it will be dedicated to the theory of phase transitions, uniqueness or non-uniqueness of the lattice Gibbs fields. We will also study the models at the criticality, where one hopes to find (in dimension 2) the onset of conformal invariance. We will see that it is indeed the case for the percolation (and the Ising model, if time permits).
The topics will include:
Crossing probabilities as a characteristic of sub-, super- and at- criticality.
Critical percolation and its power-law behavior.
The Russo-Seymour-Welsh theory of crossing probabilities - a cornerstone of critical percolation
Cardy's formula for crossing probabilities
Parafermionic observables and S. Smirnov theory
Conformal invariance of two-dimensional percolation a la Khristoforov.
Ising model in 1D, 2D and 3D.
Ising model on Cayley trees.
Conformal invariance of two-dimensional Ising model
O(N)-symmetric models
Continuous symmetry in 2D systems: The Mermin–Wagner Theorem and the absence of Goldstone bosons.
The Berezinskii–Kosterlitz–Thouless transition
Reflection Positivity and the chessboard estimates in statistical mechanics
Infrared bounds and breaking of continuous symmetry in 3D

Course Description (in Russian)

Мой курс посвящён строгим результатам статистической механики и теории просачивания. Я расскажу о некоторых элементах теории фазовых переходов в решётчатых моделях (типа модели Изинга). Значительная часть курса будет посвящена конформной инвариантности двумерной теории просачивания в критической точке и аналогичного утверждения для модели Изинга (если позволит время).
Я расскажу о поведении вероятностей наличия связи между противоположными сторонами прямоугольников (crossing probabilities) в зависимости от значения вероятности просачивания. Это поведение различает три режима: критический, надкритический и подкритический.
Центральной частью курса является изучение двумерных моделей в критическом режиме. Я расскажу очень красивое доказательство теоремы Смирнова о конформной инвариантности двумерного просачивания, найденное М. Христофоровым.
Слушатель курса научится читать и понимать статьи о конформной инвариантности в статистической физике. Он овладеет методами строгого изучения явления фазового перехода.

2. Basic Information

Course Academic Level MSc

Number of ECTS credits 6

Course Prerequisites / Recommendations

It is desirable that the students are familiar with the probability theory, measure theory, elements of functional analysis, and complex analysis. Of course, calculus knowledge is assumed.
Some familiarity with percolation theory and the Ising model will not harm, of course.

Type of Assessment Graded

Mapping from grades to percentage:

A: 85

B: 75

C: 60

D: 50

E: 40

Term Multi-term 3-4

Students of Which Programs do You Recommend to Consider this Course as an Elective?

BSc Programs	Masters Programs	PhD Programs
	Mathematical and Theoretical Physics	Mathematics and Mechanics

Maximum Number of Students

	Maximum Number of Students
Overall:	20
Per Group (for seminars and labs):	20

Course Stream Science, Technology and Engineering (STE)

3. Course Content

Topic	Summary of Topic	Contact Hours: Lectures	Contact Hours: Seminars	Contact Hours: Labs	Non-contact Hours: Student's Independent Study
Basic notions of percolation theory. Crossing probabilities.	The behavior of the probabilities of having a left-right crossing in the rectangle box. Subcritical, supercritical and critical case.	2	2	2	10
The critical value of the percolation parameter. Kesten theorem: p_c=1/2.	The value of the critical probability. Sharpness of the criticality. Power-law decay of connectivities.	2	2	2	10
The main technique of the critical percolation: the Russo-Seymour-Welsh theory	Crossing probabilities for rectangles with arbitrary aspect ratio: recent versions and proofs.	2	2	2	10
Cardy's formula	The exact formula for crossing probabilities, as correctly predicted by physicists on the basis of conformal invariance conjecture.	2	2	2	10
Parafermionic observables	The exact meaning of conformal invarinace. The key ingredients of the proof of conformal inariance.	2	2	2	10
Conformal invariance of two-dimensional percolation	The proof of the Theorem of Stanislav Smirnov -- the Khristoforov version.	2	2	2	10
Ising model on 1D, 2D and 3D lattices. Ising model on trees	Phase transitions on various lattices. Spin-glass phases.	2	2	2	10
O(N)-symmetric models	Systems with continuous symmetry in 2D and 3D. Open problems. Continuous symmetry breaking.	2	2	2	10

Topic	Summary of Topic	Contact Hours: Lectures	Contact Hours: Seminars	Contact Hours: Labs	Non-contact Hours: Student's Independent Study
The Mermin–Wagner Theorem	2D models with continuous symmetry. No Goldstone bosons in 2D. Is continuous symmetry breaking in 2D possible?	2	2	2	5
The Berezinskii–Kosterlitz–Thouless transition	Vortices and the Coulomb gase in 2D. Dipoles. Debye screening.	2	2	2	5
Reflection Positivity and the chessboard estimates. Infrared bounds	Methods of the field theory in statistical mechanics. Continuous symmetry breaking in 3D.	2	2	2	5

4. Learning Outcomes

Skoltech Learning Outcomes are indicated as per Skoltech Learning Outcomes Framework.

1. FUNDAMENTAL KNOWLEDGE

1.1. KNOWLEDGE OF MATHEMATICS AND NATURAL SCIENCES

2.1. COGNITION AND MODES OF REASONING

2.1.1. Analytical reasoning and problem solving

2.1.3. Creative thinking

2.1.5. Critical thinking and meta-cognition

2.2. ATTITUDES AND LEARNING PROCESS

2.2.5. Self-awareness and a commitment to self-improvement, lifelong learning and educating

2.3. ETHICS, EQUITY AND OTHER RESPONSIBILITIES

2.3.1. Ethical action, integrity and courage

3.1. COMMUNICATIONS IN INTERNATIONAL ENVIRONMENTS

3.1.4. Inquiry, listening and dialogue

5. Assignments and Grading

Physical Attendance Requirement (% of classes) 80

Assignment Type	Assignment Summary	% of Final Course Grade
Final Exam	Explaining the proof of a theorem from the course. General discussion of the subject covered.	40
Class participation	Asking and answering questions.	30
Homework Assignments	Solving the exersise problems and explaining the solutions in the class	30

6. Assessment Criteria

Assignment 1 Type Final Exam

Sample of Assignment 1

Explaining the proof of a theorem from the course.
General discussion of the subject covered.

Assessment Criteria for Assignment 1

Full score for the complete and correct proof.
Partial credit for an incomplete proof, depending on the understanding of the subject shown.

Assignment 2 Type Class participation

Sample of Assignment 2

1. I ask a question: can one define the conformal invariance in 3D?
2. The student asks a question: is percolation continuous in 3D?

Assessment Criteria for Assignment 2

1. A - for a meaningful discussion of the topic. No assessment otherwise.
2. A - for a meaningful discussion of the topic. Fields medal for a complete solution.

Assignment 3 Type Homework Assignments

Sample of Assignment 3

Please give a proof of the absence of breaking of the continuous symmetry in 2D systems.

Assessment Criteria for Assignment 3

A - for the correct proof.
For wrong or incomplete proof - depending on the sketch.

Assignment 4 Type Homework Assignments

Sample of Assignment 4

Describe the ground state configurations of the Ising model on Cayley tree.

Assessment Criteria for Assignment 4

A - for continuum family of ground state configurations.

For smaller families -- depending on their cardinality.

7. Textbooks and Internet Resources

You can request at most two required textbooks. Additionaly, you can suggest up to nine recommended textbooks.

Required Textbooks	ISBN-13 (or ISBN-10)
Theory of Phase Transitions: Rigorous Results by Ya. G. Sinai	9780080264691
G. Grimmett. Percolation. Springer-Verlag, Berlin, 1999.	978-3-642-08442-3

Recommended Textbooks	ISBN-13 (or ISBN-10)
Statistical Physics L D Landau E.M. Lifshitz	9780080570464

Papers	DOI or URL
Dmitry Chelkak and Stanislav Smirnov. Universality in the 2D Ising model and conformal invariance of fermionic observables. Invent. Math., 189(3):515–580, 2012.	

Web-resources (links)	Description
http://www.unige.ch/math/folks/velenik/smbook/index.html	Statistical Mechanics of Lattice Systems: a Concrete Mathematical Introduction
https://www.ihes.fr/~duminil/publi/2017percolation.pdf	Lectures on percolation theory.

8. Facilities

Software
Mathematica

Equipment
laptop

Labs for Education None

9. Additional Notes

Comments

I prefer students to ask as many questions as possible.
I do not mind taking turns and deviate from the initial plan of my lectures.

The proposed course 1) has explicit academic content and requirements for receiving credits, 2) is in alignment with the program's learning outcomes, 3) adheres to policies and Skoltech regulations.

Lead Instructor confirms