Skoltech

Course Syllabus

Course Title	Geometry in Field Theory, First Step
Course Title (in Russian)	Геометрия в теории поля, Первый Этап
Lead Instructor	Rosly, Alexey

1. Annotation

Course Description

This is a continuation of the subject started in "Some Uses of Twistors in Field Theory", 2024.

The subject of this course will be mainly twistors and the complex geometry. The word 'twistor' should be properly understood as 'twistor transform'. The latter in short means the following.

To certain 4-dimansional Riemannian (or pseudo-Riemannian, but Riemannian case is still more convenient) manifolds one can associate a 3-dimensional complex manifold. In this setting, the points of a 4-fold M are in 1:1 correspondence with "real" lines in the complex 3-fold P. For example, to the Riemannian 4-fold $M=S^4$ one gets associated complex 3-fold $P=CP^3$, while for $M=R^4=S^4\setminus\{\infty\}$ one gets $M=CP^3\setminus\{\text{a line}\}$. In this context M plays the role of a (Euclidian version of) the space-time, whereas P is called the twistor space. As a matter of fact, this construction can be carried out for certain 4n-dimensional Riemannian metrics, yielding a complex manifold the twistor space) of dimension 2n+1. This correspondence allows one to rewrite some interesting equation of Mathematical Physics for fields on M in terms of complex geometry of P. This transition can be called the twistor transform.

In this course we shall consider a number of examples of such a character.

You can find a more detailed plan in an attached file in § 9.

Course Description (in Russian)

Предмет этого курса - твисторы и комплексная геометрия. Слово "твисторы" следует понимать как более определенное понятие "твисторное преобразование". Речь о следующем.

С некоторыми 4-мерными римановыми (или псевдоримановыми, но римановы удобнее) многообразиями можно определенным образом связать 3-мерное комплексное многообразие, называемое твисторным пространством. При этом точки 4-мерного M находятся в 1:1 соответствии с "вещественными" прямыми в комплексном 3-мерном P. Например, риманову многообразию $M=S^4$ отвечает комплексное проективное пространство $P=CP^3$, а если $M=R^4=S^4\setminus\{\infty\}$, то $P=CP^3\setminus\{\text{прямая}\}$. В этом контексте M играет роль пространства-времени (в евклидовой версии), P называется твисторным пространством. На самом деле, можно рассматривать не только 4-мерный случай, а и некоторые 4n-мерные метрики, по которым строится (2n+1)-мерное комплексное (твисторное) многообразие.
Такая связь позволяет переписать некоторые интересные уравнения матфизики для полей на M в терминах комплексной геометрии на P. Этот переход можно назвать твисторным преобразованием.

В этом курсе мы рассмотрим ряд примеров такого характера.

Более детальный план - в прикрепленном файле в § 9.

2. Basic Information

Course Academic Level MSc PhD

Number of ECTS credits 6

Course Prerequisites / Recommendations

Student should be familiar with basic differential and especially complex geometry.

Type of Assessment Graded

Mapping from grades to percentage:

A: 86

B: 76

C: 66

D: 56

E: 46

F: 0

Term Multiterm (Terms 3-4)

Students of Which Programs do You Recommend to Consider this Course as an Elective?

BSc Programs	Masters Programs	PhD Programs
	Mathematical and Theoretical Physics	Mathematics and Mechanics Physics

Maximum Number of Students

	Maximum Number of Students
Overall:	10
Per Group (for seminars and labs):	10

3. Course Content

Topic	Summary of Topic	Contact Hours: Lectures	Contact Hours: Seminars	Contact Hours: Labs	Non-contact Hours: Student's Independent Study
Twistor theory	Construction of twistor transform	20	20		41

Topic	Summary of Topic	Contact Hours: Lectures	Contact Hours: Seminars	Contact Hours: Labs	Non-contact Hours: Student's Independent Study
Basics on complex geometry	Holomorphic vector bundles, related cohomology.	20	20		41

4. Learning Outcomes

Skoltech Learning Outcomes are indicated as per Skoltech Learning Outcomes Framework.

1. FUNDAMENTAL KNOWLEDGE

1.1. KNOWLEDGE OF MATHEMATICS AND NATURAL SCIENCES

5. Assignments and Grading

Physical Attendance Requirement (% of classes) 60

Assignment Type	Assignment Summary	% of Final Course Grade
Final Exam	solving problems	100

6. Assessment Criteria

Assignment 1 Type Final Exam

Assessment Criteria for Assignment 1

Solving problems from a list provided by the instructor. Students prepare solutions in advance, before the examination. They are encouraged to start solving problems during the semester. This normally leads to a higher grade because there is time to discuss their solutions and make corrections. However, all the estimates are made by the time of exam.
The main goal of the examination is to force students to consider the material of the course in all detail and to achieve a better understanding, which usually comes from "active thinking", rather than "passive". The problems to be solved for this assignment are not technically complicated and, in principle, can be found in literature. However, the main point is how a student understands what he/she has written as a solution.
This is easily seen when the discussion with the examiner on the final exam comes.

7. Textbooks and Internet Resources

You can request at most two required textbooks. Additionaly, you can suggest up to nine recommended textbooks.

Required Textbooks	ISBN-13 (or ISBN-10)
some textbook	

Recommended Textbooks	ISBN-13 (or ISBN-10)
Principles of Algebraic Geometry. Author(s):. Phillip Griffiths, Joseph Harris	9780471050599
М.Атья, Геометрия полей Янга-Миллса, В: М.Атья, Геометрия узлов и физика, Москва, «Мир», 1995	5-03-002892-7
R.Ward, Integrable Systems and Twistors In: Integrable Systems: Twistors, Loop Groups, and Riemann Surfaces (Oxford Graduate Texts in Mathematics, Vol. 4) (Oxford Graduate Texts in Mathematics (No. 4)) 1st Edition, by N. J. Hitchin, G. B. Segal, R. S. Ward	978-0199676774

8. Facilities

9. Additional Notes

The proposed course 1) has explicit academic content and requirements for receiving credits, 2) is in alignment with the program's learning outcomes, 3) adheres to policies and Skoltech regulations.

Lead Instructor confirms