Skoltech

Course Syllabus

Course Title	Integrable Systems of Classical Mechanics
Course Title (in Russian)	Интегрируемые системы классической механики
Lead Instructor	Prokofev, Vadim

1. Annotation

Course Description

The course is devoted to the study of integrable systems, their general properties and methods applied to their study them. We will use the example of integrable systems of classical mechanics. Using the example of such models as the Calogero-Moser, Ruijenaars-Schneider, Toda systems, we will talk about the Lax representation, r-matrix, Bäcklund transformations - methods used for integrable systems in general. In addition, attention will be paid to the relationship of integrable systems with each other and with other branches of mathematical physics.

Course Description (in Russian)

Курс посвящен изучению интегрируемых систем, их общих свойствах и методов, применяющихся к их изучению на примере интегрируемых систем классической механики. На примере таких моделей как системы Калоджеро-Мозера, Руйсенарса-Шнайдера, Тоды, будет рассказано о представлении Лакса, r-матрицы, преобразованиях Бэклунда -- методов используемых для интегрируемых систем в целом. Помимо этого будет уделено внимание взаимосвязи интегрируемых систем друг с другом и с другими разделами математической физики.

2. Basic Information

Course Academic Level MSc PhD

Number of ECTS credits 6

Type of Assessment Graded

Mapping from grades to percentage:

A: 86

B: 76

C: 66

D: 56

E: 46

F: 0

Term Term 1-2

Students of Which Programs do You Recommend to Consider this Course as an Elective?

BSc Programs	Masters Programs	PhD Programs
	Mathematical and Theoretical Physics	Mathematics and Mechanics

Maximum Number of Students

	Maximum Number of Students
Overall:	10
Per Group (for seminars and labs):	10

3. Course Content

Topic	Summary of Topic	Contact Hours: Lectures	Contact Hours: Seminars	Contact Hours: Labs	Non-contact Hours: Student's Independent Study
Rational and hyperbolic Calogero-Moser systems.	Formulation of rational and hyperbolic Calogero-Moser system and their algebraic solution. Toda system as limit of hyperbolic Calogero System and its generalization	10			25

Topic	Summary of Topic	Contact Hours: Lectures	Contact Hours: Seminars	Contact Hours: Labs	Non-contact Hours: Student's Independent Study
Trigonometric and elliptic Calogero-Moser System. rational Calogero-Moser system in quadratic potential	Algebraic solutions of trigonometric CM system and rational CM system in quadratic potential, equilibrium positions, small oscillations. Elliptic functions, general solution of functional equation.	10			25
Root System and Coxeter groups. Generalized CM systems	Introducing Coxter groups and root systems. Classifications of finite Root systems. Construction of Generalized CM system	10			15
P-Q duality and Ruijsenaars-Schneider systems	P-Q selfduality of rational CM system. Rational Ruijsenaars-Schneider system as P-Q dual to Hyperbolic CM system. RS systems as "relativistic" generalization of CM systems.	8			15
Backlund transformation	General definition of Backlund transformation. Backlund transformation for CM and RS systems.	6			20
Hydrodynamical Limit of CM systems	Hydrodynamical limit of dynamical system. Benjamin Ono equation as hydrodynamical limit of CM equation	6			12

4. Learning Outcomes

Skoltech Learning Outcomes are indicated as per Skoltech Learning Outcomes Framework.

1. FUNDAMENTAL KNOWLEDGE

1.1. KNOWLEDGE OF MATHEMATICS AND NATURAL SCIENCES

2.1. COGNITION AND MODES OF REASONING

2.1.2. System thinking

2.1.3. Creative thinking

2.2. ATTITUDES AND LEARNING PROCESS

2.2.6. Development and support of teaching and learning community

5. Assignments and Grading

Physical Attendance Requirement (% of classes) 70

Assignment Type	Assignment Summary	% of Final Course Grade
Exercise	Set of problems for each topic	80
Final Project	A topic close to course material for student to study by himself.	20

6. Assessment Criteria

Assignment 1 Type Exercise

Sample of Assignment 1

Consider an equation of motion for rational Calogero system with two particles and solve it.

Assessment Criteria for Assignment 1

+ will be given for complete solution without errors.
+- will be given for error calculating integral.
-+ will be given if student will use central of mass reference frame to provide further calculations but the rest of the solution turn out to be wrong

Assignment 2 Type Final Project

Sample of Assignment 2

Analogically to Calogero-Moser model find functional equation, which describe all possible potentials of Ruijsenaars-Schneider model. Find all possible solutions. Generalize result for different types of root systems.

These types of tasks are only for students who wants a maximal grade. Maximal number of points can be given if student completes task and can answer some questions about his solution during conversation. Some errors are acceptable without lowering mark if student can correct or identify them during conversation.

7. Textbooks and Internet Resources

You can request at most two required textbooks. Additionaly, you can suggest up to nine recommended textbooks.

Required Textbooks	ISBN-13 (or ISBN-10)
А.М. Переломов, Интегрируемые системы классической механики и алгебры Ли, Москва, Наука, 1990	5-02-013826-6

Papers	DOI or URL
"Quantum versus classical integrability in Calogero-Moser systems" Edward Corrigan, R. Sasaki J.Phys.A 35 (2002) 7017-7062	https://doi.org/10.1088/0305-4470/35/33/306

8. Facilities

9. Additional Notes

The proposed course 1) has explicit academic content and requirements for receiving credits, 2) is in alignment with the program's learning outcomes, 3) adheres to policies and Skoltech regulations.

Lead Instructor confirms