Skoltech

Course Syllabus

Course Title Hamiltonian Mechanics

Course Title (in Russian) Гамильтонова механика

Lead Instructor Poberezhny Vladimir

Co-Instructor

First Name	Last Name
Alexey	Basalaev

1. Annotation

Course Description

Hamiltonian mechanics is one of basic disciplines in mathematical physics. The scope of our course is to introduce students to modern views on the fundamentals of the theory of integrable systems and mathematical physics. Mastering its program makes it possible to study of advanced courses of mathematical physics.
The mathematics of the modern theory of Hamiltonian systems includes methods of the theory of differential equations and dynamical systems, Lie groups and algebras and their representations, symplectic and Poisson geometry, analysis on manifolds and many others. Acquiring practical skills in applying the techniques and structures of these domains of mathematics, the ability to combine them to solve problems in mechanics is one of the main goals of this course. The course can be recommended not only to students of mathematical physics program, but also to those planning to specialize in pure mathematics or its applications.

Course Description (in Russian)

Гамильтонова механика относится к базовым матфизическим дисциплинам. Наш курс направлен на знакомство слушателей с современными взглядами на основы теории интегрируемых систем и математической физики. Освоение его программы даёт возможность в дальнейшем изучать более продвинутые курсы, связанные с математической физикой.
Математический аппарат современной теории гамильтоновых систем включает в себя методы теории дифференциальных уравнений и динамических систем, групп и алгебр Ли и их представлений, симплектической и пуассоновой геометрии, анализа на многобразиях и многих других. Приобретение практических навыков применения приёмов и конструкций этих разделов математики, умение их сочетать для решения задач механики является одной из основных целей данного курса. Курс может быть рекомендован не только студентам-матфизикам, но и планирующим специализироваться в чистой математике или её приложениях.

2. Basic Information

Course Academic Level BSc MSc

Number of ECTS credits 6

Course Prerequisites / Recommendations

Standard courses of Real analysis, differential equations and analysis on manifolds/

Type of Assessment Graded

Mapping from grades to percentage:

A: 86

B: 76

C: 66

D: 56

E: 46

F: 0

Term Terms 1-2

Students of Which Programs do You Recommend to Consider this Course as an Elective?

BSc Programs	Masters Programs	PhD Programs
	Mathematical and Theoretical Physics	Mathematics and Mechanics

Maximum Number of Students

	Maximum Number of Students
Overall:	10
Per Group (for seminars and labs):	10

3. Course Content

Topic	Summary of Topic	Contact Hours: Lectures	Contact Hours: Seminars	Contact Hours: Labs	Non-contact Hours: Student's Independent Study
Newtonian formalism	Geometry of Newtonian mechanics. Symmetries, conservation laws and integrability in Newtonian context	8	8	0	16

Topic	Summary of Topic	Contact Hours: Lectures	Contact Hours: Seminars	Contact Hours: Labs	Non-contact Hours: Student's Independent Study
Lagrangian formalism	Geometry of Lagrangian mechanics. Symmetries, conservation laws and integrability in Lagrangian context	8	8	0	16
Hamiltonian formalism	Geometry,symmetries, conservation laws. Liouville-Arnold integrability. Hamilton-Jacobi equation. Separation of variables.	8	8	0	16
Symplectic and Poisson structures	Darboux theorem, canonical structures, systems on Lie algebras, coadjoint orbits, Kirillov-Costant bracket	8	8	0	16
Hamiltonian reduction	Moment maps, homogenious spaces and hamiltonian reduction	8	10	0	16

4. Learning Outcomes

Skoltech Learning Outcomes are indicated as per Skoltech Learning Outcomes Framework.

1. FUNDAMENTAL KNOWLEDGE

1.1. KNOWLEDGE OF MATHEMATICS AND NATURAL SCIENCES

1.4. INTERDISCIPLINARY THINKING, KNOWLEDGE STRUCTURE AND INTEGRATION

2.1. COGNITION AND MODES OF REASONING

2.1.1. Analytical reasoning and problem solving

2.1.2. System thinking

2.1.3. Creative thinking

5. Assignments and Grading

Physical Attendance Requirement (% of classes) 75

Assignment Type	Assignment Summary	% of Final Course Grade
Final Exam	Oral exam on theory with short problem solving part	50
Problem Set	Set of problems covering first half of the course	25
Problem Set	Set of problems covering second half of the course	25

6. Assessment Criteria

Assignment 1 Type

Problem Set

Sample of Assignment 1

On two-dimensional plane a particle of mass m moves under the action of force $F=\partial_v$, where coordinates (x,y) are Carthesian and coordinates $(x,y)=(u\cos v, 2u\sin v)$. Derive equations of motion and find the solution with given initial data.

Assessment Criteria for Assignment 1

Highest number of points will be given for obtaining equations of motion in explicit form and correct calculation of parameters corresponding to initial data. Incorrect calculation of solution will reduce the points by 25%. Incorrect applicatiopn of Newton law to derive the equation reduse the points by 75%.

Assignment 2 Type

Final Exam

Sample of Assignment 2

Give the definition of coadjoint action and construct the Kirillov-Costant bracket on its orbits.

Full mark is given if the student completly understand the topic and can give clear and profound explanation of all constructions used in his answer.

Assignment 3 Type

Problem Set

Sample of Assignment 3

Prove that Poisson bracket of two integrals of motion explicitely dependent on time is again an integral of motion.

Assessment Criteria for Assignment 3

Incorrect choice of hamiltonian equation of motion for the function explicitely depending on time reduce the points by 75%.

7. Textbooks and Internet Resources

You can request at most two required textbooks. Additionaly, you can suggest up to nine recommended textbooks.

Required Textbooks	ISBN-13 (or ISBN-10)
A.M. Perelomov, Integrable Systems of Classical Mechanics and Lie Algebras, Birkhäuser Basel, 1990	978-3-7643-2336-3
V.I. Arnold, Mathematical Methods of Classical Mechanics, Springer New York, NY	978-1-4757-1693-1

Recommended Textbooks	ISBN-13 (or ISBN-10)
Guillemin, Sternberg, Symplectic methods in physics, Cambridge University Press	9780521389907

Papers	DOI or URL
B. A. Dubrovin, I. M. Krichever, S. P. Novikov, “Integrable systems. I”, Dynamical systems – 4, Itogi Nauki i Tekhniki. Ser. Sovrem. Probl. Mat. Fund. Napr., 4, VINITI, Moscow, 1985, 179–277	https://www.mathnet.ru/php/getFT.phtml?jrnid=intf&paperid=37&what=fullt&option_lang=eng

8. Facilities

Labs for Education

NRU HSE-Skoltech International Laboratory of Representation Theory and Mathematical Physics

9. Additional Notes

The proposed course 1) has explicit academic content and requirements for receiving credits, 2) is in alignment with the program's learning outcomes, 3) adheres to policies and Skoltech regulations.

Lead Instructor confirms