Skoltech

Course Syllabus

Course Title	Classical groups, their invariants and representations
Course Title (in Russian)	Классические группы, их инварианты и представления
Lead Instructor	Olshanski, Grigori

Co-Instructor

First Name	Last Name
Alexei	Illin

1. Annotation

Course Description

The title of the course is deliberately copied from the famous book by Hermann Weyl (1939; 1946). The material in the book forms the core of representation theory. For this reason, working through this material is useful for everyone who wants to deal with any problems in representation theory or apply its results. The purpose of the course is to introduce students to the main ideas and results of Weyl's book, as well as to their further development. Of course, in addition to Weyl’s book, we will use other, more modern sources.

Tentative program:

1. Four series A, B, C, D of complex classical groups. Compact classical groups. Classical Lie algebras.

2. Center of universal enveloping algebra and Harish-Chandra's homomorphism.

3. Capelli identity.

4. Invariant theory for complex classical groups: various versions of the first fundamental theorem.

5. Haar measure on compact classical groups, its radial part (Weyl's formula).

6. Irreducible characters: first and second Weyl's formulas.

7. Realization of fundamental representations.

8. Polynomial representations and Schur-Weyl duality.

9. Binomial formula for characters and interpolation Schur polynomials.

10. Character branching rules.

11. Universal characters of Koike-Terada.

12. Weyl duality in traceless tensors.

13. Brauer duality.

Course Description (in Russian)

Название курса намеренно скопировано со знаменитой книги Германа Вейля (1939; 1946). Материал книги составляет ядро теории представлений. Поэтому проработать этот материал полезно всем, кто хочет заниматься какими бы то ни было задачами теорией представлений или применять ее результаты. Цель курса --- познакомить студентов с основными идеями и результатами книги Вейля, а также и их дальнейшим развитием. Разумеется, помимо книги Вейля, мы будем использовать и другие, более современные источники.

Примерная программа:

1. Четыре серии A,B,C,D комплексных классических групп. Компактные классические группы. Классические алгебры Ли.

2. Центр универсальной обертывающей алгебры и гомоморфизм Хариш-Чандры.

3. Тождество Капелли.

4. Теория инвариантов для комплексных классических групп: различные версии первой основной теоремы.

5. Мера Хаара на компактных классических группах, ее радиальная часть (формула Вейля).

6. Неприводимые характеры: первая и вторая формулы Вейля.

7. Реализация фундаментальных представлений.

8. Полиномиальные представления и двойственность Шура-Вейля.

9. Биномиальная формула для характеров и интерполяционные полиномы Шура.

10. Правила ветвления для характеров.

11. Универсальные характеры Койке-Терады.

12. Двойственность Вейля в бесследовых тензорах.

13. Двойственность Брауэра.

2. Basic Information

Course Academic Level

MSc PhD

Number of ECTS credits

6

Course Prerequisites / Recommendations

Algebra and linear algebra (compulsory courses of the first two years). Familiarity with the basics of the theory of Lie groups and Lie algebras is highly desirable (courses on this topic were regularly taught in the first semester at the Faculty of Mathematics of the HSE University).

Type of Assessment

Graded

Mapping from grades to percentage:

A: 86

B: 76

C: 66

D: 56

E: 46

F: 0

Term

Term 1 Term 2

Students of Which Programs do You Recommend to Consider this Course as an Elective?

BSc Programs	Masters Programs	PhD Programs
	Mathematical and Theoretical Physics	Mathematics and Mechanics

Maximum Number of Students

	Maximum Number of Students
Overall:	20
Per Group (for seminars and labs):	20

3. Course Content

Topic	Summary of Topic	Contact Hours: Lectures	Contact Hours: Seminars	Contact Hours: Labs	Non-contact Hours: Student's Independent Study
Preliminaries	Structure of classical Lie algebras and groups	8	8	0	24
Invariants	Description of invariants for classical group actions in tensors and other natural spaces	8	8	0	24
Representations	Parametrization of irreducible representations by highest weights. Character formulas	8	8	0	25
Complementary material	Interpolation polynomials, Weyl duality, Brauer duality	8	8	0	25

4. Learning Outcomes

Skoltech Learning Outcomes are indicated as per Skoltech Learning Outcomes Framework.

1. FUNDAMENTAL KNOWLEDGE

1.1. KNOWLEDGE OF MATHEMATICS AND NATURAL SCIENCES

5. Assignments and Grading

Physical Attendance Requirement (% of classes) 80

Assignment Type	Assignment Summary	% of Final Course Grade
Homework Assignments	---	100

6. Assessment Criteria

Assignment 1 Type Homework Assignments

Sample of Assignment 1

1. Describe the root system and the triangular decomposition for each series of classical complex Lie algebras.

2. Find generators in the algebra of invariants for various classical group actions. E.g. for the adjoint representation.

3. Find decomposition on irreducible components for concrete representations.

4. Compute eigenvalues of concrete central elements of universal enveloping algebras.

Assessment Criteria for Assignment 1

The exercises will vary in difficulty. The total score is calculated according to the formula min(100, 200*S/N), where S denotes the total number of points obtained and N denotes the maximal possible

number of points.

7. Textbooks and Internet Resources

You can request at most two required textbooks. Additionaly, you can suggest up to nine recommended textbooks.

Required Textbooks	ISBN-13 (or ISBN-10)
--	---

Recommended Textbooks	ISBN-13 (or ISBN-10)
H. Weyl. The Classical Groups. Their Invariants and Representations. Amer. Math. Soc., 1939; 1946. W. Fulton, J. Harris. Representation Theory, a First Course (Graduate Texts in Mathematics 129), Springer-Verlag, New York, 1991. D. P. Zelobenko. Compact Lie Groups and Their Representations. Translations of Mathematical Monographs, Vol. 40, American Mathematical Society, Providence, R.I., 1973. R. Goodman, N. R. Wallach. Symmetry, Representations, and Invariants. Springer, 2009. C. Procesi. Lie groups. An Approach through Invariants and Representations. Springer, 2007.	

Papers	DOI or URL
---	---

8. Facilities

Labs for Education n/a

9. Additional Notes

The proposed course 1) has explicit academic content and requirements for receiving credits, 2) is in alignment with the program's learning outcomes, 3) adheres to policies and Skoltech regulations.

Lead Instructor confirms