Skoltech

Course Syllabus

Course Title	Introduction in Algebraic Topology
Course Title (in Russian)	Введение в алгебраическую топологию
Lead Instructor	Kazarian, Maxim

1. Annotation

Course Description

Algebraic topology is one of the most striking subjects than makes mathematics of XX century dramatically different from all previous mathematics. It becomes nowadays a necessary part of contemporary mathematical study. A combination of geometric ideas with formalized computational algorithms make an efficient tool for studies of many mathematical structures, including those which are not related directly to topology. The course will introduce to the basic notions of algebraic topology, including homology and cohomology. We will pay attention to both rigorous formulation of main theorems of the theory and also to the development of computational skills allowing one to get explicit numerical answer to practical problems appearing in applications.

Course Description (in Russian)

Алгебраическая топология является одним из ярчайших предметов, которые отличают математику XX века от всей предшествующей. В настоящее время она является незаменимым инструментом современных математических исследований. Комбинация геометрических идей с явными формализованными алгоритмами в этой области являются эффективным средством для исследования многих математических структур, в том числе и не связанных напрямую с топологией. В курсе мы дадим основные понятия алгебраической топологии, включая гомологии и когомологии. Упор будет делаться как на строгие формулировки основных теорем, так и на выработку техники для решения практических задач, возникающих в приложениях, и доведения ответа "до числа"

2. Basic Information

Course Academic Level MSc PhD

Number of ECTS credits 6

Course Prerequisites / Recommendations

Familiarity with basic notions of topology including fundamental group is desirable.

Type of Assessment Graded

Mapping from grades to percentage:

A: 86

B: 76

C: 66

D: 56

E: 46

F: 0

Term Multiterm 1-2

Students of Which Programs do You Recommend to Consider this Course as an Elective?

BSc Programs	Masters Programs	PhD Programs
	Mathematical and Theoretical Physics	Mathematics and Mechanics

Maximum Number of Students

	Maximum Number of Students
Overall:	20
Per Group (for seminars and labs):	20

3. Course Content

Topic	Summary of Topic	Contact Hours: Lectures	Contact Hours: Seminars	Contact Hours: Labs	Non-contact Hours: Student's Independent Study
Topological spaces	Operations with topological spacees, a cone, suspension, join	4	4		10

Topic	Summary of Topic	Contact Hours: Lectures	Contact Hours: Seminars	Contact Hours: Labs	Non-contact Hours: Student's Independent Study
Chain complexes	Cycles, boundaries, homological equivalence, homology	4	4		10
Simplicial homology	Simplicial space and its chain complex, the boundary operator	4	4		10
Cellular and simplicial homology	different approaches to Construction of homology, equivalence for cellular spaces	4	4		10
Exact long sequence	Applications to computations of homology of certain spaces	4	4		10
Homology of manifolds	Fundamental class, orientability, Poincare duality	4	4		10
Intersection index and degree of a mapping	Transversality, local intersection index, classification of maps of spheres of equal dimensions, linking number	4	4		10
Cohomology and multiplication	Cup product and intersection of cycles, computstion of cohomology ring of projective spaces	4	4		10
Morse theory	Morse index, Morse comples, Morse inequalities	4	4		10

4. Learning Outcomes

Skoltech Learning Outcomes are indicated as per Skoltech Learning Outcomes Framework.

1. FUNDAMENTAL KNOWLEDGE

1.1. KNOWLEDGE OF MATHEMATICS AND NATURAL SCIENCES

2.1. COGNITION AND MODES OF REASONING

2.1.1. Analytical reasoning and problem solving

2.2. ATTITUDES AND LEARNING PROCESS

2.2.4. Resourcefulness, flexibility and an ability to adapt

2.2.5. Self-awareness and a commitment to self-improvement, lifelong learning and educating

2.3. ETHICS, EQUITY AND OTHER RESPONSIBILITIES

2.3.1. Ethical action, integrity and courage

2.3.4. Trust and loyalty

2.3.6. Commitment to social and professional behavior

3.1. COMMUNICATIONS IN INTERNATIONAL ENVIRONMENTS

3.1.2. Written, electronic and graphical communication

3.1.3. Oral presentation and discussion

3.1.5. Communications in English in scientific, business and social settings

3.2. TEAMWORK AND LEADERSHIP

3.2.5. Technical and multidisciplinary teaming

3.3. COLLABORATION AND CHANGE

3.3.1. Establishing diverse connections and networking

4.1. MAKING SENSE OF GLOBAL SOCIETAL ENVIRONMENTAL AND BUSINESS CONTEXT

4.1.1. Appreciating the potential and limitations of science and technology, their role in society and society's role in their evolution

4.1.3. Understanding the technical products, systems and infrastructure of the sector

4.2. VISIONING — INVENTING NEW TECHNOLOGIES THROUGH RESEARCH

4.2.1. The research process — hypothesis, evidence and defense

4.2.2. Basic research leading to new scientific discovery

4.3. VISIONING — CONCEIVING AND DESIGNING SUSTAINABLE SYSTEMS

4.3.1. Identifying stakeholders need and wants

4.3.2. Identifying and formulating objectives and goals

5. Assignments and Grading

Physical Attendance Requirement (% of classes) 80

In-person Attendance Requirement Comment

students missing the classes should to present in written form all problems and theorems discussed in the classes

Assignment Type	Assignment Summary	% of Final Course Grade
Exercise	Classroom activity and written hometasks	20
Midterm Exam	Written test with 3-5 problems of different levels of diiculty	40
Final Exam	Written test with 3-5 problems of different levels of diiculty	40

6. Assessment Criteria

Assignment 1 Type Final Exam

Sample of Assignment 1

1 Determine the Euler characteristic of the following spaces\\
a) $\R^n$;\\
b) the space of (nonoriented) affine lines in $\R^3$;\\
c) the space of (nonoriented) projective lines in $\R P^3$;\\
d) the space of symmetric nondegenerate 2×2 matrices.

2 The topological space X is obtained from the disjoint union of a circle~S^1 and a cylinder $S^1\times[0,1]$ by gluing along a map of the boundary $f:\partial(S^1\times[0,1])\to S^1$ such that the restriction of f to the boundary components $S^1\times \{0\}$ and $S^1\times\{1\}$ have the degrees m and n, respectively. Compute the (integer) homology of~X. Which pairs of parameter values (m,n) give rise to homotopy equivalent spaces?

3 The standardly embedded surfaces $\R P^2\subset\C P^2$ and $\C P^1\subset\C P^2$ represent two homology classes in $H_2(\C P^2,\Z_2)$. Are these classes equal?

4 Let $X_1=S^4$, $X_2=S^2\times S^2$, $X_3=\C P^2$. For which pairs (i,j) there exists a continuous map $X_i\to X_j$ of degree 2? If the map exists, give an example. If it does not, prove it.

5 The parametrization of the sphere in the spherical coordinates (ϕ,θ) is given by
$$x=\cos\phi\cos\theta, y=\cos\phi\sin\theta, z=\sin\phi.$$
Consider a mapping $S^2\to S^2$ given by (ϕ,θ) by $\phi\mapsto n\phi$, $\theta\mapsto m\theta$. Compute the degree of this map in the following cases:\\
(a) $n=0$;\\
(b) $n=2$, $m=0$;\\
(c) any m and n.

Or Upload Sample of Assignment 1

Sample of Assignment 1

Assessment Criteria for Assignment 1

The grade is formed by the percentage of the solved problems, up to 10 points for each solved problem.

Assignment 2 Type Midterm Exam

Sample of Assignment 2

1. Let $\gamma:[0,1]\to X$ be a continuous path connecting points $a=\gamma(0)$ and $b=\gamma(1)$ and considered as a singular 1-simplex. Denote by $\overline\gamma$ the reversed path, $\overline\gamma(t)=\gamma(1-t)$. Check that the singular 1-chain $a=\gamma+\overline\gamma$ satifies $\partial a=0$. Construct a singular 2-chain b such that $\partial b = a$.

\medskip
2. Compute integer homology of the following spaces:

a) S^3 without standardly embedded (unknotted) circle;

b) two copies of $\R P^2$ glued along a circle embedded to each of the two copies of the projective plane

as a projective line;

c) two copies of the sphere S^2 glued along a circle embedded to each of the two copies of the sphere as an equator;

\medskip
3. Compute the degree of the map $\C P^2\to\C P^2$ given in homogeneous coordinates by $$(x:y:z)\mapsto (x^3:y^3:z^3).$$

\medskip
4. Does there exist a continuous mapping f of the wedge product $S^2\vee S^2$ to itself such that it is not homotopic to identity but its third iteration $f\circ f\circ f$ is homotopic to identity?

The grade is formed by the percentage of the solved problems, up to 10 points for each solved problem.

7. Textbooks and Internet Resources

You can request at most two required textbooks. Additionaly, you can suggest up to nine recommended textbooks.

Required Textbooks	ISBN-13 (or ISBN-10)
A. Hatcher. Algebraic topology	0521795400, 9780521795401

Recommended Textbooks	ISBN-13 (or ISBN-10)
D. B. Fuks, A. T. Fomenko, Homotopic topology	9783319234885, 3319234889

8. Facilities

9. Additional Notes

The proposed course 1) has explicit academic content and requirements for receiving credits, 2) is in alignment with the program's learning outcomes, 3) adheres to policies and Skoltech regulations.

Lead Instructor confirms