Skoltech

Course Syllabus

Course Title	Characteristic Classes
Course Title (in Russian)	Характеристические классы
Lead Instructor	Gaifullin, Alexander

1. Annotation

Course Description

The course will include introduction to theory of characteristic classes, namely, the Stiefel-Whitney, Chern and Pontryagin classes of vector bundles and the Miller-Morita-Mumford classes of fibre bundles with fibre an oriented surface. The exposition will be based on the splitting principle and on the computation of the cohomology rings of the Grassmann manifolds. We will also discuss the connection with theory of cohomological operations (Steenrod squares), including the formulae due to Wu and Thom.

Course Description (in Russian)

Курс будет включать в себя введение в теорию характеристических классов, а именно, классов Штифеля-Уитни, Черна и Понтрягина векторных расслоений и характеристических классов Миллера-Мориты-Мамфорда локально тривиальных расслоений со слоем ориентированная поверхность. Изложение будет основано на принципе расщепления, а также на вычислении колец когомологий грассмановых многообразий. Кроме того, мы обсудим связь теории характеристических классов с теорией когомологических операций (квадратов Стинрода), включая формулы Ву и Тома.

2. Basic Information

Course Academic Level

MSc PhD

Number of ECTS credits

6

Course Prerequisites / Recommendations

Students should be familiar with standard courses of calculus, linear algebra, differential geometry, and an introductory course of topology.

Type of Assessment

Graded

Mapping from grades to percentage:

A: 86

B: 76

C: 66

D: 56

E: 46

F: 0

Term

Term1-2

Students of Which Programs do You Recommend to Consider this Course as an Elective?

BSc Programs	Masters Programs	PhD Programs
	Mathematical and Theoretical Physics	Mathematics and Mechanics Physics

Maximum Number of Students

	Maximum Number of Students
Overall:	30
Per Group (for seminars and labs):	30

3. Course Content

Topic	Summary of Topic	Contact Hours: Lectures	Contact Hours: Seminars	Contact Hours: Labs	Non-contact Hours: Student's Independent Study
Homology and cohomology	CW complexes. Cell decompositions of smooth manifolds corresponding to Morse functions. Cell decompositions of projective spaces and Grassmannians. Sard's theorem. Degree of a map. Cellular approximation theorem. Homology and cohomology. Steenrod-Eilenberg axioms. Mayer-Vietoris sequence. Computation of the homology of Grassmannians. De Rham cohomology. De Rham theorem. Multiplication in cohomology. Poincare duality. Intersection of cycles.	10	10		46
Fibre bundles and classifying spaces.	Fibre bundles. Vector and principal bundles. Associated bundles. Serre fibrations. Thom space of a vector bundle. Thom class. Thom isomorphism. Euler class. Leray-Hirsch theorem. Construction of Chern and Stiefel-Whitney characteristic classes. Splitting principle. Classifying spaces of Lie groups (topological groups). Characteristic classes as cohomology classes of classifying spaces. Classifying spaces for classical Lie groups. General construction of classifying spaces. Classifying spaces of discrete groups. Eilenberg-MacLane spaces. Classifying spaces of braid groups.	10	10		46
K-theory	K-groups of a topological space. Bott periodicity theorem. K-theory as a cohomology theory. Thom class in K-theory of complex vector bundles. Pushforward homomorphism. Riemann-Roch-Hirzebruch-Grothendieck theorem. 12) Класс Тома комплексных расслоений в K-теории. Гомоморфизм прямого образа. Теорема Римана -- Роха -- Хирцебруха -- Гротендика.	5	5		20

4. Learning Outcomes

Skoltech Learning Outcomes are indicated as per Skoltech Learning Outcomes Framework.

1. FUNDAMENTAL KNOWLEDGE

1.1. KNOWLEDGE OF MATHEMATICS AND NATURAL SCIENCES

2.1. COGNITION AND MODES OF REASONING

2.1.1. Analytical reasoning and problem solving

2.1.3. Creative thinking

2.1.4. Decision making (with ambiguity, urgency etc.)

2.2. ATTITUDES AND LEARNING PROCESS

2.2.2. Willingness to make decisions in the face of uncertainty

2.2.3. Responsibility, intensity, perseverance, urgency and will to deliver

2.2.4. Resourcefulness, flexibility and an ability to adapt

2.2.5. Self-awareness and a commitment to self-improvement, lifelong learning and educating

2.2.6. Development and support of teaching and learning community

2.3. ETHICS, EQUITY AND OTHER RESPONSIBILITIES

2.3.6. Commitment to social and professional behavior

3.1. COMMUNICATIONS IN INTERNATIONAL ENVIRONMENTS

3.1.2. Written, electronic and graphical communication

3.1.3. Oral presentation and discussion

3.1.4. Inquiry, listening and dialogue

3.1.5. Communications in English in scientific, business and social settings

3.2. TEAMWORK AND LEADERSHIP

3.2.1. Forming effective teams

3.3. COLLABORATION AND CHANGE

3.3.1. Establishing diverse connections and networking

4.1. MAKING SENSE OF GLOBAL SOCIETAL ENVIRONMENTAL AND BUSINESS CONTEXT

4.1.1. Appreciating the potential and limitations of science and technology, their role in society and society's role in their evolution

4.2. VISIONING — INVENTING NEW TECHNOLOGIES THROUGH RESEARCH

4.2.1. The research process — hypothesis, evidence and defense

4.2.2. Basic research leading to new scientific discovery

4.3. VISIONING — CONCEIVING AND DESIGNING SUSTAINABLE SYSTEMS

4.3.2. Identifying and formulating objectives and goals

5. Assignments and Grading

Physical Attendance Requirement (% of classes) 80

Assignment Type	Assignment Summary	% of Final Course Grade
Final Exam		100

6. Assessment Criteria

Assignment 1 Type Final Exam

Sample of Assignment 1

Compute the first Pontryagin class of the quaternionic projective plane HP(2).

Assessment Criteria for Assignment 1

The maximal number of points can be achieved by correct and precise computation based on theorems and techniques from the course, with a correct answer. Either algebraic topological or differential geometric approaches can be used. Various reduction can be made for various mistakes, including inaccuracy with non-commutativity of quaternions.

7. Textbooks and Internet Resources

You can request at most two required textbooks. Additionaly, you can suggest up to nine recommended textbooks.

Required Textbooks	ISBN-13 (or ISBN-10)
Milnor, JW, Morse theory	0-691-08008-9
Bott, R, Tu, LW, Differential forms in algebraic topology	9780387906133

Recommended Textbooks	ISBN-13 (or ISBN-10)
Milnor, JW, Stasheff, JD. Characteristic classes	978-0691081229
Atiyah, M, K-Theory	978-0201407921
Hatcher, Algebraic Topology	978-0521795401
Fomenko, AT, Fuchs, DB, Homotopical topology	978-3-319-23487-8
Schwarz, AS, Quantum Field Theory and Topology	978-3540547532

8. Facilities

9. Additional Notes

The proposed course 1) has explicit academic content and requirements for receiving credits, 2) is in alignment with the program's learning outcomes, 3) adheres to policies and Skoltech regulations.

Lead Instructor confirms