Skoltech

Course Syllabus

Course Title	Geometric Representation Theory
Course Title (in Russian)	Геометрическая теория представлений
Lead Instructor	Finkelberg, Mikhail
Co-Instructor	Finkelberg, Mikhail

1. Annotation

Course Description

Geometric representation theory applies algebraic geometry to the problems of representation theory. Some of the most famous problems of representation theory were solved on this way during the last 40 years. The list includes the Langlands reciprocity for the general linear groups over the functional fields, the Langlands-Shelstad fundamental Lemma, the proof of the Kazhdan-Lusztig conjectures; the computation of the characters of the finite groups of Lie type. We will study representations of the affine Hecke algebras using the geometry of affine Grassmannians (Satake isomorphism) and Steinberg varieties of triples (Deligne-Langlands conjecture).

This is a course for master students knowing the basics of algebraic geometry, sheaf theory, homology and K-theory.

Course Description (in Russian)

Геометрическая теория представлений применяет алгебраическую геометрию к теории представлений. Одни из самых важных и глубоких задач теории представлений были решены на этом пути в течении последних сорока лет. Этот список включает взаимность Ленглендса для полных линейных групп над функциональными полями, фундаментальную лемму Ленглендса-Шельстад, доказательство гипотез Каждана-Люстига, вычисление характеров конечных групп лиевского типа. Мы будем изучать представления аффинных алгебр Гекке, используя геометрию аффинных Грассманнианов (изоморфизм Сатаке) и многообразий троек Стейнберга (гипотеза Делиня-Ленглендса).

Это курс для магистрантов и аспирантов, владеющих основами алгебраической геометрии, теории пучков, гомологий и К-теории. В процессе освоения этого курса слушатели узнают об извращённых пучка и овладеют понятием гиперболических слоёв.
Прослушавшие курс студенты будут уметь вычислять близкие и исчезающие циклы.

2. Basic Information

Course Academic Level

MSc PhD

Number of ECTS credits

6

Course Prerequisites / Recommendations

The basic algebraic geometry, sheaf theory, homology and K-theory.

Type of Assessment

Graded

Mapping from grades to percentage:

A: 80

B: 70

C: 60

D: 50

E: 40

F: 0

Term

Term 1-2

Students of Which Programs do You Recommend to Consider this Course as an Elective?

BSc Programs	Masters Programs	PhD Programs
	Mathematical and Theoretical Physics	Mathematics and Mechanics Physics

Maximum Number of Students

	Maximum Number of Students
Overall:	20
Per Group (for seminars and labs):	20

Course Stream

Science, Technology and Engineering (STE)

3. Course Content

Topic	Summary of Topic	Contact Hours: Lectures	Contact Hours: Seminars	Contact Hours: Labs	Non-contact Hours: Student's Independent Study
Affine Grassmannians	Schubert varieties of finite and infinite codimension, semiinfinite orbits.	10	10	10	10
Hyperbolic stalks	Dimension estimates for the intersection of semiinfinite and G(O)-orbits. Exactness of the hyperbolic stalks.	10	10	10	10
Convolution	Exactness of convolution. Convolution vs. fusion. Commutativity constraint.	10	10	10	10
Kazhdan-Lusztig-Ginzburg construction	Demazure operators in the equivariant K-theory of the Steinberg triple variety. Relation of Borel-Moore homology and Ext-algebra for semismall resolutions.	10	10	10	12

4. Learning Outcomes

Skoltech Learning Outcomes are indicated as per Skoltech Learning Outcomes Framework.

1. FUNDAMENTAL KNOWLEDGE

1.1. KNOWLEDGE OF MATHEMATICS AND NATURAL SCIENCES

2.1. COGNITION AND MODES OF REASONING

2.1.1. Analytical reasoning and problem solving

2.2. ATTITUDES AND LEARNING PROCESS

2.2.2. Willingness to make decisions in the face of uncertainty

2.3. ETHICS, EQUITY AND OTHER RESPONSIBILITIES

2.3.5. Proactive vision and intention in life

3.1. COMMUNICATIONS IN INTERNATIONAL ENVIRONMENTS

3.1.3. Oral presentation and discussion

3.2. TEAMWORK AND LEADERSHIP

3.2.1. Forming effective teams

3.3. COLLABORATION AND CHANGE

3.3.1. Establishing diverse connections and networking

4.1. MAKING SENSE OF GLOBAL SOCIETAL ENVIRONMENTAL AND BUSINESS CONTEXT

4.1.1. Appreciating the potential and limitations of science and technology, their role in society and society's role in their evolution

4.2. VISIONING — INVENTING NEW TECHNOLOGIES THROUGH RESEARCH

4.2.1. The research process — hypothesis, evidence and defense

4.3. VISIONING — CONCEIVING AND DESIGNING SUSTAINABLE SYSTEMS

4.3.2. Identifying and formulating objectives and goals

4.4. DELIVERING ON THE VISION — IMPLEMENTING AND OPERATING

4.4.4. Implementation and operations management

4.5. DELIVERING ON THE VISION — ENTREPRENEURSHIP AND ENTERPRISE

4.5.7. Managing intellectual property and respecting the legal process

5. Assignments and Grading

Physical Attendance Requirement (% of classes) 80

Assignment Type	Assignment Summary	% of Final Course Grade
Homework Assignments	weekly home assignments	80
Final Exam	final exam	20

6. Assessment Criteria

Assignment 1 Type Problem Set

Sample of Assignment 1

1. Prove that the IC sheaves with complex coefficients of the G(O)-orbit closures in the affine Grassmannian of GL(2) are constant.
2. Find the IC stalks with coefficients in the algebraic closure of a finite field of characteristic p of the G(O)-orbit closures in the affine Grassmannian of GL(2).
3. Find the IC stalks of the minimal G(O)-orbit closure in the affine Grassmannian of a simple algebraic group G.
4. Find the hyperbolic stalks in the problem 3 above.
5. Find the usual and hyperbolic stalks of the IC sheaf of the nilpotent cone of a simple Lie algebra g.

Assessment Criteria for Assignment 1

1. Correct proof: 10 points; incorrect proof: 0 points.
2. Correct answer for all p: 10 points; correct answer for p>2: 8 points; correct answer for p=infty: 5 points.
3. Correct answer for all G: 10 points; correct answer for classical G: 5 points.

4. Correct answer for all G: 10 points; correct answer for classical G: 5 points.
5. Correct answer for all g: 10 points; correct answer for classical g: 5 points.

Assignment 2 Type Final Exam

Sample of Assignment 2

1. Compute the equivariant Hilbert polynomial of the orbital varieties of the Lie algebra sl(3).

Correct answer for all the orbital varieties: 10 points; correct answer for the principal variety: 5 points.

7. Textbooks and Internet Resources

You can request at most two required textbooks. Additionaly, you can suggest up to nine recommended textbooks.

Required Textbooks	ISBN-13 (or ISBN-10)
Representation theoryand complex geometry.Chriss N., Ginzburg V., Birkhauser, Boston, 2010.	9780817649371

Recommended Textbooks	ISBN-13 (or ISBN-10)
Macdonald I.G., Symmetric functions and Hall algebras, Clarendon Press, 2015.	9780198739128

8. Facilities

9. Additional Notes

The proposed course 1) has explicit academic content and requirements for receiving credits, 2) is in alignment with the program's learning outcomes, 3) adheres to policies and Skoltech regulations.

Lead Instructor confirms