Skoltech

Course Syllabus

Course Title	Cluster Integrable Systems
Course Title (in Russian)	Кластерные интегрируемые системы
Lead Instructor	Bershtein, Mikhail
Co-Instructor	Litvinov, Alexey; Marshakov, Andrey

1. Annotation

Course Description

Cluster integrable systems form a relatively new, interesting, and important class of integrable systems. One of their basic features is that they are multiplicative (in more physical words relativistic). Another important feature is the natural constructions of discrete flows and quantization. Perhaps the most important application is the (conjectural) structure of cluster integrable system on the Coulomb branches of 4d supersymmetric theories.

In the course we will discuss basic examples of cluster integrable systems. Familiarity with cluster algebras and varieties is not assumed; indeed, the course is designed as an introduction to these concepts.

The course will be based on the works of Fock, Goncharov, Marshakov, Kenyon, Schrader, Shapiro.

Course Description (in Russian)

Кластерные интегрируемые системы представляют собой относительно новый, интересный и важный класс интегрируемых систем. Одно из их основных свойст -- это то, что они являются мультипликативными (в более физических терминах релятивистскими). Другая важная особенность - естественные конструкции дискретных потоков и квантования. Возможно, наиболее важным приложением является (предположительная) структура кластерной интегрируемой системы на кулоновских ветвях 4d-суперсимметричных теорий.

На курсе мы обсудим основные примеры кластерных интегрируемых систем. Знакомство с кластерными алгебрами и многообразиями не предполагается, наоборот курс может рассматриваться как введение в эти понятия.

Курс основан на работах Фока, Гончарова, Маршакова, Кеньона, Шрадера, Шапиро.

2. Basic Information

Course Academic Level

BSc MSc PhD

Number of ECTS credits

6

Course Prerequisites / Recommendations

Basic knowledge of differential geometry is necessary. Some experience with integrable systems and Lie groups is highly recommended. Some acquaintance with local systems and quantum groups could be useful.

Type of Assessment

Graded

Mapping from grades to percentage:

A: 86

B: 76

C: 66

D: 56

E: 46

F: 0

Term

Multiterm 1-2

Students of Which Programs do You Recommend to Consider this Course as an Elective?

BSc Programs	Masters Programs	PhD Programs
	Mathematical and Theoretical Physics	Mathematics and Mechanics

Maximum Number of Students

	Maximum Number of Students
Overall:	10
Per Group (for seminars and labs):	10

3. Course Content

Topic	Summary of Topic	Contact Hours: Lectures	Contact Hours: Seminars	Contact Hours: Labs	Non-contact Hours: Student's Independent Study
Introduction to integrable systems	Poisson brackets, classical integrable systems. Example of open Toda system.	2	2	2	8
Poisson-Lie groups	Poisson-Lie groups. Sklyanin bracket. Double Bruhat cells	2	2	2	8
Cluster varieties	Cluster varieties. Seeds, mutations.	2	2	2	8
Open relativistic Toda integrable system	Amalgamation. Cluster structures on double Bruhat cells. Quotient by Cartan.	2	2	2	8
Global functions	A variables. Starfish lemma. Global functions.	2	2	2	8
Loop groups	Double Bruhat cells in affine group. Closed Toda system.	2	2	2	8
Dimer partition function on torus.	Consistent graphs on torus, Kasteleyn orientations, Zigzags. Goncharov-Kenyon integrable systems. Newton Polygons. Discrete flows	4	4	4	12
Moduli spaces of decorated local system on surface with punctures	General definition. Dehn twist. Functions on G and G^*	4	4	4	14
Quantization	Quantization of cluster varieties. Functions on the quantum group. Quantum group,	2	2	2	8
Quantum Toda system	Quantum Toda system. Baxter operator. Miniscule operators.	2	2	2	8

4. Learning Outcomes

Skoltech Learning Outcomes are indicated as per Skoltech Learning Outcomes Framework.

1. FUNDAMENTAL KNOWLEDGE

1.1. KNOWLEDGE OF MATHEMATICS AND NATURAL SCIENCES

2.1. COGNITION AND MODES OF REASONING

2.1.3. Creative thinking

2.2. ATTITUDES AND LEARNING PROCESS

2.2.3. Responsibility, intensity, perseverance, urgency and will to deliver

2.2.4. Resourcefulness, flexibility and an ability to adapt

4.2. VISIONING — INVENTING NEW TECHNOLOGIES THROUGH RESEARCH

4.2.1. The research process — hypothesis, evidence and defense

4.2.2. Basic research leading to new scientific discovery

5. Assignments and Grading

Physical Attendance Requirement (% of classes) 50

Assignment Type	Assignment Summary	% of Final Course Grade
Problem Set	Problem Set 1	40
Problem Set	Problem Set 2	40
Problem Set	Problem Set 3	20

6. Assessment Criteria

Assignment 1 Type Problem Set

Sample of Assignment 1
Find symplectic leafs for SL(2) group with Sklyanin Poisson bracket

Assessment Criteria for Assignment 1
Proportional to the number of solved problems.

Assignment 2 Type Problem Set

Sample of Assignment 2
Find Hamiltonian for the Goncharov-Kenyon integrable system with square Newton polygon

Proportional to the number of solved problems.

Assignment 3 Type Problem Set

Sample of Assignment 3
Show commutativity of Baxter operators for open Toda system

Assessment Criteria for Assignment 3
Proportional to the number of solved problems.

7. Textbooks and Internet Resources

You can request at most two required textbooks. Additionaly, you can suggest up to nine recommended textbooks.

Required Textbooks	ISBN-13 (or ISBN-10)
Sergey Fomin, Lauren Williams, Andrei Zelevinsky Introduction to Cluster Algebras.	

Recommended Textbooks	ISBN-13 (or ISBN-10)
Michael Gekhtman, Michael Shapiro, and Alek Vainshtein Cluster Algebras and Poisson Geometry	978-0-8218-4972-9

Papers	DOI or URL
Fock V., Goncharov A. Cluster X-varieties, amalgamation and Poisson-Lie groups	https://doi.org/10.1007/978-0-8176-4532-8_2
Fock V., Goncharov A. Moduli spaces of local systems and higher Teichmuller theory	https://doi.org/10.1007/s10240-006-0039-4
Goncharov, A.B., Kenyon R. Dimers and cluster integrable systems	https://doi.org/10.24033/asens.2201
Fock, V.V., Marshakov, A. Loop Groups, Clusters, Dimers and Integrable Systems	doi.org/10.1007/978-3-319-33578-0_1
Schrader G. Shapiro A On b-Whittaker functions	

Web-resources (links)	Description
https://dept.math.lsa.umich.edu/~fomin/cluster.html	Cluster Algebras Portal by Sergei Fomin

8. Facilities

9. Additional Notes

The proposed course 1) has explicit academic content and requirements for receiving credits, 2) is in alignment with the program's learning outcomes, 3) adheres to policies and Skoltech regulations.

Lead Instructor confirms