Geometry and Integrability
Conference program

Monday, 18 September

Place: Skoltech, space E-B1-2016

11:00 – 12:00	Maxim Kazarian, "Topological recursion, xy duality, and KP integrability"
12:00 – 12:30	*Coffee break*
12:30 – 13:30	Huijun Fan, "LG/CY correspondence between tt^*-geometries"
14:00 – 15:00	*Lunch*
15:00 – 16:00	Iskander Taimanov, "Geometry of the Moutard transformations for Dirac operators"
16:00 – 16:20	*Coffee break*
16:20 – 17:20	Xiaobo Liu, "Tautological Relations and Their Applications"
17:30 – 18:30	Michael Shapiro (online), "Log-canonical coordinates for symplectic groupoid of unipotent upper-triangular matrices and Teichmüller space of closed genus two surfaces"

Tuesday, 19 September

Place: Skoltech, space E-B1-2016

11:00 – 12:00	Gerard Helminck, "A construction of solutions of an integrable deformation of a commutative Lie algebra of skew hermitian $\mathbb{Z} \times \mathbb{Z}$-matrices"
12:00 – 12:30	*Coffee break*
12:30 – 13:30	Ismagil Habibullin, "Classification of integrable lattices in 3D via Darboux integrable 2D reductions"
14:00 – 15:00	*Lunch*
15:00 – 16:00	Alexander Mikhailov, "Quantisation ideals: multi-quantum systems and systems with non-deformation quantisation"
16:00 – 16:20	*Coffee break*
16:20 – 17:20	Rustem Garifullin, "Integrable discrete equations with unusual properties"
17:30 – 18:30	Alexandr Popolitov, "On monomial matrix models' superintegrability"

Wednesday, 20 September

Place: Faculty of Mathematics (HSE University), room 108

11:00 – 12:00	Shuai Guo, "Virasoro constraints and polynomial recursion"
12:00 – 12:20	*Coffee break*
12:20 – 13:20	Di Yang, "Mapping Partition Functions and the Mapping Universality Conjecture"
13:20 – 14:50	*Lunch*
14:50 – 15:50	Chenglang Yang, "Action of W-type operators on Schur Q-functions, and their application"

Thursday, 21 September

Place: Faculty of Mathematics (HSE University), room 110

11:00 – 12:00	Dimitry Gurevich, "Reflection Equation Algebras and related combinatorics"
12:00 – 12:20	*Coffee break*
12:20 – 13:20	Sotiris Konstantinou-Rizos, "N-simplex maps on groups and rings"
13:20 – 14:50	*Lunch*
14:50 – 15:50	Victor Mishnyakov, "Commutative subalgebras of the Affine Yangian $Y(gl_1)$ and matrix models"
15:50 – 16:10	*Coffee break*
16:10 – 17:10	Xavier Blot, "Hierarchies of meromorphic differentials"

17:20 – 18:20	Raphaël Fesler, “Ribbon gluing and real Hurwitz numbers”

Friday, 22 September

Place: Skoltech, space E-B1-2016

11:00 – 12:00	Sergey Derkachov, “Open Toda chain: Q-operator, orthogonality and completeness of eigenfunctions”
12:00 – 12:30	*Coffee break*
12:30 – 13:30	Nikita Belousov, “Bispectrality in Ruijsenaars hyperbolic model”
14:00 – 15:00	*Lunch*
15:00 – 16:00	Dmitry Talalaev, “The full symmetric Toda system, quantization, solution via QR decomposition and geometry of the flag varieties”

Abstracts

Nikita Belousov

Bispectrality in Ruijsenaars hyperbolic model

Bispectrality is a situation when a wave function of a quantum model, apart from solving some equations with respect to (say) coordinates, also solves some equations with respect to spectral parameters. In the talk I will explain how this situation is realized in the integrable Ruijsenaars hyperbolic model and how it can be used to study the properties of its wave functions.

Xavier Blot

Hierarchies of meromorphic differentials

I will present a new construction of integrable hierarchies based on the strata of meromorphic differentials. Although the construction works for any Cohomological Field Theory (CohFT) in both the classical and quantum settings, I will focus on constructing the classical hierarchy associated with the trivial CohFT. Then, I will provide results on these hierarchies, explaining their relation to the KdV hierarchy and, more generally, to the DR hierarchies. This work also yields new results on intersection numbers on the strata of differentials. This is a joint work with Paolo Rossi.

Sergey Derkachov

Open Toda chain: Q-operator, orthogonality and completeness of eigenfunctions

We consider the open Toda chain as illustrative example to demonstrate the main steps in construction of eigenfunctions in some quantum integrable models and proof of their orthogonality and completeness.

Huijun Fan

LG/CY correspondence between tt^*-geometries

tt^*-geometry structure was found by physicists in the 1980's, and defined and developed later in mathematics at the beginning of 90's. It is an integrable structure mixed with the holomorphic and anti-holomorphic parts, and has close connections with Higgs bundles, Frobenius manifolds and other interesting structures. It is believed that it can be applied to more important occasions. The tt^* geometrical structures of Calabi–Yau manifolds have been built long ago in the name of "special geometry". In this talk, I will explain my construction of tt^*-geometry for Landau–Ginzburg model via geometrical analysis method long time ago and formulate very recent results building the explicit LG/CY isomorphism between tt^* geometrical structures for projective CY hypersurfaces. The latter work appears in arxiv:2210.16747.

Raphaël Fesler

Ribbon gluing and real Hurwitz numbers

Classical (or complex) Hurwitz numbers have many definitions, among them there is a topological one as the number of ways to glue ribbons to a collection of disks so as to obtain a given surface with boundary. A purely algebraic one as the number of ways to multiply transpositions so as to obtain a permutation of a given cyclic type; and an algebro-geometric one as the number non isomorphic ramified cover over the sphere, where all the critical values are simple except possibly over infinity. Furthermore the generating function of the Hurwitz numbers satisfies the cut-and-join equation and can be explicitly expressed via Schur polynomials. In this talk we will discuss these definitions as well as their real analogs. This talk is based on a joint work with Yu. Burman

Rustem Garifullin

Integrable discrete equations with unusual properties

We demonstrate a number of integrable discrete equations of the form

$$F_{n,m}(u_{n,m}, u_{n+1,m}, u_{n,m+1}, u_{n+1,m+1}) = 0, \quad n, m \in \mathbb{Z},$$

where $u_{n,m}$ is an unknown function, n and m are integer independent variables, F is a given function of two discrete and 4 continuous variables. According to the structure of higher symmetries and first integrals, these equations are discrete analogues of hyperbolic type equations

$$u_{xy} = f(u_x, u_y, u, x, y).$$

It is shown that the equations under consideration have unusual properties in terms of the structure of their generalized symmetries and Lax pairs. So, in particular, some equations have different order of generalized symmetries in different directions. Examples of equations for which the order of simplest generalized symmetry can be arbitrarily large are given.

Shuai Guo

Virasoro constraints and polynomial recursion

Virasoro constraints are a hypothetical framework that arises in many enumerative geometry problems. In this talk, we will investigate the properties and applications of the Virasoro constraints for all genera. First, we will derive the ancestor form of the Virasoro constraints, which leads to a polynomial recursion relation. For semisimple cases, this recursion completely determines the generating series of higher genus invariants, extending Gathmann's result. Then, we will propose a generalized Virasoro conjecture for the CohFTs with non-flat units. For semisimple theories, we will prove this conjecture by using the Givental–Teleman reconstruction theorem. This talk is based on joint work with Qingsheng Zhang.

Dimitry Gurevich

Reflection Equation Algebras and related combinatorics

REA are particular cases of the so-called Matrix Quantum algebras. However, they are of special interest since they admit introducing analogs of certain symmetric functions with remarkable properties. I plan to develop some elements of the corresponding combinatorics and exhibit some applications of these algebras to integrable system theory.

Ismagil Habibullin

Classification of integrable lattices in 3D via Darboux integrable 2D reductions

In our recent articles, we have shown that nonlinear integrable equations of the Toda lattice type

$$u^j_{x,y} = f(u^j_x, u^j_y, u^{j+1}, u^j, u^{j-1}),$$

as well as integrable semidiscrete models with two discrete and one continuous independent variables of the following form

$$u^j_{n+1,x} = f(u^j_{n,x}, u^{j+1}_n, u^j_n, u^j_{n+1}, u^{j-1}_{n+1})$$

and, finally, integrable equations of the Hirota-Miwa type

$$u^j_{n+1,m+1} = f(u^{j+1}_{n+1,m}, u^j_{n+1,m}, u^j_{n,m}, u^j_{n,m+1}, u^{j-1}_{n,m+1})$$

have the following property. All of them are reduced to systems of equations of hyperbolic type, integrable in the sense of Darboux, differential, differential-difference and, accordingly, completely discrete, by imposing at the points $j = 0$ and $j = N$ with an arbitrary $N > 0$ the conditions of termination of some special kind.

The results of our investigations convincingly show that the existence of such reductions is, in fact, a criterion for the integrability of a nonlinear lattice.

This property of nonlinear lattices can be successfully used to construct localized particular solutions, as well as in solving the problem of classification of integrable lattices with three independent variables. It is well known that a system of equations of hyperbolic type is Darboux integrable, i.e. admits a complete set of characteristic integrals, if and only if its characteristic

Lie-Rinehart algebras in both directions are of finite dimension. We also proposed an algorithm for classifying integrable hyperbolic systems of exponential type using characteristic algebras.

Gerard Helminck

A construction of solutions of an integrable deformation of a commutative Lie algebra of skew hermitian $\mathbb{Z} \times \mathbb{Z}$-matrices

Inside the algebra $LT_{\mathbb{Z}}(R)$ of $\mathbb{Z}\times\mathbb{Z}$-matrices with coefficients from a commutative $\mathbb{C}$-algebra R that have only a finite number of nonzero diagonals above the central diagonal, we consider a deformation of a commutative Lie algebra $\mathcal{C}_{sh}(\mathbb{C})$ of finite band skew hermitian matrices. The evolution equations that the deformed generators of $\mathcal{C}_{sh}(\mathbb{C})$ have to satisfy are determined by the decomposition of $LT_{\mathbb{Z}}(R)$ in the direct sum of an algebra of lower triangular matrices and the finite band skew hermitian matrices. This yields then the $\mathcal{C}_{sh}(\mathbb{C})$-hierarchy. We show that the projections of a solution satisfy zero curvature relations and that it suffices to solve an associated Cauchy problem. Solutions of this type can be obtained by finding appropriate vectors in the $LT_{\mathbb{Z}}(R)$-module of oscillating matrices, the so-called wave matrices, that satisfy a set of equations in the oscillating matrices, called the linearization of the $\mathcal{C}_{sh}(\mathbb{C})$-hierarchy. Finally, a Hilbert Lie group will be introduced from which wave matrices for the $\mathcal{C}_{sh}(\mathbb{C})$-hierarchy are constructed.

Maxim Kazarian

Topological recursion, xy duality, and KP integrability

Topological recursion is an inductive procedure allowing one starting from a relatively small amount of initial data to compute the so called potential, i.e. a generating series whose coefficients carry one or another enumerative information. It was observed last years that the technology of topological recursion covers a huge amount of various seemingly unrelated problems in combinatorics, mathematical physics, Gromov–Witten theory, intersection theory on moduli spaces, and other domains of mathematics. The computed potential along with relations of topological recursion possesses in many cases other integrable properties. In particular, it often satisfies equations of Kadomtsev–Petviashvili integrable hierarchy. We explain in the talk the source of these integrability properties and prove, thereby, the KP integrability of many important potentials of topological recursion. The talk is based on a series of papers written jointly with A. Alexandrov, B. Bychkov, P. Dunin-Barkowski, and S. Shadrin.

Sotiris Konstantinou-Rizos

N-simplex maps on groups and rings

By N-simplex maps we mean set-theoretical solutions to the N-simplex equations, which are fundamental equations of Mathematical Physics. The most famous members of the family of n-simplex equations are: the Yang–Baxter equation ($n = 2$), the Zamolodchikov tetrahedron equation ($n = 3$) and the Bazhanov–Stroganov equation ($n = 4$). In this talk, I will present new methods for constructing 3- and 4-simplex maps. I will demonstrate the role of correspondences for deriving new n-simplex maps which do not belong to any of the known classification lists. Next, I will present some new tetrahedron maps on groups and rings. Moreover, I will show a method for constructing nontrivial 4-simplex extensions of tetrahedron maps. Finally, I will present some classification results.

Xiaobo Liu

Tautological Relations and Their Applications

Relations among tautological classes on moduli spaces of stable curves have important applications in cohomological field theory. For example, relations among psi-classes and boundary classes give universal equations for generating functions of Gromov–Witten invariants of all compact symplectic manifolds. In this talk, I will talk about such relations and their applications to Gromov–Witten theory and integrable systems.

Alexander Mikhailov

Quantisation ideals: multi-quantum systems and systems with non-deformation quantisation

We propose to reformulate the problem of quantisation, focussing on quantisation of dynamical systems themselves, rather than of their Poisson structures. We begin with a dynamical system defined on a free associative algebra $\mathcal{A}$ with non-commutative dynamical variables (often such system can be obtained as a lift of classical dynamical systems to a free algebra). The dynamical system defines a derivation $\partial : \mathcal{A} \mapsto \mathcal{A}$. Then we reduce the problem of quantisation to the problem of description of two-sided quantisation ideals, i.e. such ideals $\mathcal{J} \subset \mathcal{A}$ that are ∂–stable $\partial\mathcal{J} \subset \mathcal{J}$, and the quotient algebra $\mathcal{A}/\mathcal{J}$ admits a basis of normally ordered monomials. Quantum multiplication rules in the quotient algebra over a quantisation ideal are manifestly associative and consistent with the dynamics. The new approach also sheds light on the problem of operator's ordering.

We found first examples of bi-quantum systems which are quantum counterparts of bi-Hamiltonian systems in the classical theory. Moreover, the new approach enables us to define and present first examples of non-deformation quantisations of dynamical systems, i.e. quantum systems that can be presented in the Heisenberg form $\partial(a) = \frac{i}{\hbar}[H, a]$, but the algebra of observables remains non-commutative for any choice of "Planck's constant" $\hbar$. In the quantum deformation case the limit $\hbar \to 0$ results in a classical Poisson structure and Hamilton equations on a Poisson manifold, while in the non-deformation case such a limit yields a Poisson structure and Hamilton equations associated with the corresponding non-commutative algebra.

This talk is based on joint works (published or yet in preparation) with V. M. Buchstaber, S. Carpentier, P. Vanhaecke and J. P. Wang.

Victor Mishnyakov

Commutative subalgebras of the Affine Yangian $Y(gl_1)$ and matrix models

We explain that the study of the connection between W-representations of matrix models and their superintegrability naturally leads to commutative subalgebras of the W_∞ algebra. These subalgebras correspond to new integrable systems generalizing the Calogero family. We construct many such subalgebras and explain how they look in various representations. We explain that some of the subalgebras survive the β-deformation, an intermediate step from W_∞ to the affine Yangian. This deformation is natural on the side of many-body systems, where it actually introduces the coupling. The hidden symmetry given by the families of commuting Hamiltonians is in charge of the special, (skew) hypergeometric τ-functions.

Alexandr Popolitov

On monomial matrix models' superintegrability

I will talk about what is currently known about superintegrability of monomial matrix models (MMMs) in pure phase: the already well-known linear superintegrability results, the recently developed formulas in the exotic sector and the currently work-in-progress bilinear superintegrability in the framework of K_δ operators.

Michael Shapiro

Log-canonical coordinates for symplectic groupoid of unipotent upper-triangular matrices and Teichmüller space of closed genus two surfaces

We described a system of log-canonical coordinates for the symplectic groupoid of unipotent upper-triangular matrices equipped with the natural Poisson structure. As a byproduct, we discovered a system of log-canonical coordinates and a corresponding cluster structure for the Teichmüller space of closed genus two curves. In particular, we obtain a cluster description of the mapping class group action. This is a joint work with L. Chekhov.

Iskander Taimanov

Geometry of the Moutard transformations for Dirac operators

We expose the Moutard transformation for two-dimensional Dirac operators, explain their geometrical meanings via the spinor representations of surface in three- and four-spaces, and give some applications to constructing solutions of two-dimensional soliton equations.

Dmitry Talalaev

The full symmetric Toda system, quantization, solution via QR decomposition and geometry of the flag varieties

The full symmetric Toda system is a generalization of an open Toda chain, which is one of the archetypal examples of integrable systems. The open Toda chain illustrates a connection of the theory of integrable systems with the theory of Lie algebras and Lie groups, is a representative of the Adler–Kostant–Symes scheme for constructing and solving such systems. Until recently, only some of the results from this list were known for the full Toda system. I will talk about the construction, the commutative family, quantization and solution of the system by the QR decomposition method, as well as about the application of this system to the geometry of real flag vaireties. The material of my talk is based on several joint works with A. Sorin, Yu. Chernyakov and G. Sharygin.

Di Yang

Mapping Partition Functions and the Mapping Universality Conjecture

This is a joint work with Don Zagier. Let G be the infinite group of formal power series of one variable near the identity with group law given by composition. We introduce the G-action on partition functions of Witten–Kontsevich (WK) type. In particular, each element of G sends the WK partition function to a new partition function, which we call the WK mapping partition function. We show that the genus zero part of its logarithm is independent of elements of G, and derive the loop equation for its higher genus parts. We then show that the Dubrovin–Zhang hierarchy for the WK mapping partition function is a bihamiltonian perturbation of the Riemann–Hopf hierarchy possessing a tau-structure, and conjecture that it is a universal object for all such perturbations. A particular example of our study leads to the Hodge–WK correspondence, whose consequences give an accurate identification between matrix gravity and topological gravity.

Chenglang Yang

Action of W-type operators on Schur Q-functions, and their application

W-type operators appear in realizations of some infinite dimensional Lie algebras, offering symmetries for both the KP and BKP hierarchies. For examples, they give Virasoro operators, cut-and-join operators, and operators in the W-constraints. Schur Q-functions give characters for irreducible projective representations of symmetric groups. In this talk, I will introduce a formula for action of W-type operators on Schur Q-functions. Application to study the expansion of Brezin–Gross–Witten model in terms of Schur Q-functions will also be introduced. This talk is based on joint work with Professor Xiaobo Liu.