Course Title (in English)	Cohomology of Groups and Classifying Spaces
Course Title (in Russian)	Когомологии групп и классифицирующие пространства
Lead Instructor	Gaifullin, Alexander

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Course Description

The course will include an introduction to theory of cohomology of groups from geometric viewpoint. The concept of a classifying space of a group will be central in this course. We will start with general algebraic definitions and theorems concerning homology and cohomology of (discrete) groups. Then we proceed with various constructions of classifying spaces of groups and methods for computing their cohomology. An important part of the course will be devoted to examples such as certain finite groups, braid groups, Coxeter groups, mapping class groups, etc.

Course Prerequisites / Recommendations	Students should be familiar with the basics of algebraic topology including fundamental group, singular homology and cohomology.

Аннотация

Курс будет посвящен введению в теорию когомологий групп с геометрической точки зрения. Центральным в курсе будет понятие классифицирующего пространства группы. Мы начнем с общих алгебраических определений и теорем, касающихся гомологий и когомологий (дискретных) групп. После этого будут рассмотрены различные конструкции классифицирующих пространств групп и методы вычисления их когомологий. Важной частью курса будет изучение примеров, таких как некоторые конечные группы, группы кос, группы Кокстера, группы классов отображений и т.д.

Course Academic Level	Master-level course suitable for PhD students
Number of ECTS credits	6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Chain complexes	Chain complexes. Chain homotopy. Tensor product and Hom functors. Their derived functors. Künneth formula Projective modules. Projective resolutions.	1	6	
Group homology and cohomology	Definition of (co)homology of groups. The first and the second homology and cohomology groups. Multiplication in cohomology.	1	6	
Classifying spaces.	Intro to classifying spaces. Milnor's and Milgram's constructions for BG. Cohomology of BG as characteristic classes of G-bundles. Examples.	1	6	
Coxeter groups	Intro to Coxeter groups. CAT(0)-complexes with actions of Coxeter groups. Classifying spaces and cohomology of Coxeter groups.	2	8	
Braid groups	Classifying spaces and cohomology for pure braid group and braid group. Arnold-Thom-Pham conjecture for Artin groups.	2	6	
Lyndon-Hochschild-Serre spectral sequence.	Double complex. Spectral sequence of the double complex. LHS spectral sequence.	1	6	
Further examples: finite groups, arithmetic groups, mapping class groups.	Some further results on cohomology of several important classes of groups.	1	4	

Assignment Type	Assignment Summary
Final Exam	The exam for each student will consist of the combination of theoretical questions and solving problems on various topics in the course.

Type of Assessment	Graded
Grade Structure	<table><tr><th>Activity Type</th><th>Activity weight, %</th></tr><tr><td>Final Exam</td><td>100</td></tr></table>
A:	86
B:	76
C:	66

D:	56
E:	46
F:	0
Attendance Requirements	Mandatory with Exceptions

Maximum Number of Students

	Maximum Number of Students
Overall:	40
Per Group (for seminars and labs):	20

Course Stream	Science, Technology and Engineering (STE)
Course Term (in context of Academic Year)	Term 1-2
Course Delivery Frequency	Every two years

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Mathematical and Theoretical Physics	Mathematics and Mechanics Physics

Course Tags	Math

Required Textbooks	ISBN-13 (or ISBN-10)
Brown, KS. Cohomology of Groups. Graduate Texts in Mathematics, 87. Springer, 1982.	0387906886

Recommended Textbooks	ISBN-13 (or ISBN-10)
Evens, L. The Cohomology of Groups. Oxford University Press, 1991.	0198535805
Daverman, RJ, Sher, RB (eds). Handbook of Geometric Topology, Elsevier, 2002.	0444824324

Knowledge
After course completion students will know the basics of theory of group cohomology and classifying spaces, including particular results on several important classes of groups such as braid groups, Coxeter groups, mapping class groups, certain finite groups.

Skill
After course completion students will get skills of computations concerning cohomology of groups, including the usage of the Lyndon-Hochschild-Serre spectral sequence and several other related spectral sequences.

Experience
An important feature of theory of cohomology of groups is its deep relationships with many different areas of mathematics such as algebraic topology, differential geometry, metric geometry, low-dimensional topology, modular forms, etc. Students will get experience of applications of ideas and methods from various branches of mathematics.

Select Assignment 1 Type	Final Exam
Input Example(s) of Assignment 1 (preferable)	The final exam for each student will consist of 2 theoretical questions and 3 problems. An example of a theoretical question is: Construction of a CAT(0) complex on which the given right-angled Coxeter group acts freely. An example of a problem is: Compute the ring of integral cohomology of the symmetric group of degree 3.
Assessment Criteria for Assignment 1	Each theoretical question and each problem will constitute 20% of the total exam mark.