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Skolkovo Institute of Science and Technology

Course Title (in English)	Universal enveloping algebras and Yangians
Course Title (in Russian)	Универсальные обертывающие алгебры и янгианы
Lead Instructor	Olshanski, Grigori
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Course Description

Tentative program of the course:

1. Reminder: the classification of irreducible finite-dimensional representations of a reductive Lie algebra $\mathfrak g$; the universal enveloping algebra $U(\mathfrak g)$; the PBW theorem.

2. The structure of the center $Z(U(\mathfrak g))$; the Harish-Chandra map.

3. The symmetrization map $S(\mathfrak g)\to U(\mathfrak g)$ and related combinatorics

4. The Perelomov--Popov formula (mainly for $\mathfrak g=\mathfrak{gl}(N,\mathbb C)$; for other classical Lie algebras --- without proof)

5. The Capelli identity

6. Quantum immanants (after Okounkov)

7. The Yangian for $\mathfrak g=\mathfrak{gl}(N,\mathbb C)$ and the R-matrix formalism

8. The twisted Yangians

9. Works of Gelfand--Kirillov

Course Prerequisites / Recommendations	Good working knowledge of the university course of algebra, including linear algebra. It is highly desirable to know the fundamentals of the theory of Lie groups and Lie algebras, up to the classification of irreducible finite-dimensional representations of the Lie algebra gl(N).

Аннотация

Примерная программа курса:

1. Напоминания: классификация неприводимых конечномерных представлений редуктивной алгебры Ли $\mathfrak g$; универсальная обертывающая алгебра $U(\mathfrak frak g)$; теорема ПБВ.

2. Структура центра $Z(U(\mathfrak g))$; отображение Хариш-Чандры.

3. Отображение симметризации $S(\mathfrak g)\to U(\mathfrak g)$ и связанная с ним комбинаторика.

4. Формула Переломова-Попова (в основном, для $\mathfrak g=\mathfrak{gl}(N,\mathbb C)$; для других классических алгебр Ли --- без доказательства)

5. Тождество Капелли.

6. Квантовые имманенты (по Окунькову)

7. Янгиан для $\mathfrak g=\mathfrak{gl}(N,\mathbb C)$ и R-матричный формализм.

8. Скрученные янгианы.

9. Работы Гельфанда--Кириллова.

Course Academic Level	Master-level course suitable for PhD students

		Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Topic	Summary of Topic			
Universal enveloping algebras: basic facts	Reminder: the classification of irreducible finite-dimensional representations of a reductive Lie algebra $\mathfrak g$; the universal enveloping algebra $U(\mathfrak g)$; the PBW theorem. The structure of the center $Z(U(\mathfrak g))$; the Harish-Chandra map. The symmetrization map $S(\mathfrak g)\to U(\mathfrak g)$ and related combinatorics	9	7	
The case of $U(gl(N,C))$	Perelomov-Popov formula, Capelli identities, and other results related to $U(gl(N,C))$	9	7	
The Yangians and the R-matrix formalism.	The Yangian of the Lie algebra gl(N,C). Its center, automorphisms, the Hopf structure. The twisted Yangians.	10	7	

Assignment Type	Assignment Summary
Homework Assignments	sets of exercises for each lecture

Number of ECTS credits   6

Type of Assessment	Graded

Grade Structure

Activity Type	Activity weight, %
Homework Assignments	100

A:	86
B:	76
C:	66
D:	56
E:	46
F:	0
Attendance Requirements	Optional with Exceptions
Maximum Number of Students	

	Maximum Number of Students
Overall:	20
Per Group (for seminars and labs):	20

Course Stream	Science, Technology and Engineering (STE)

Course Term (in context of Academic Year)	Term 1 Term 2

Course Delivery Frequency	n/a

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Mathematical and Theoretical Physics	Mathematics and Mechanics

Course Tags	Math

Required Textbooks	ISBN-13 (or ISBN-10)

Recommended Textbooks	ISBN-13 (or ISBN-10)
Alexander Molev. Yangians and classical Lie algebras, AMS 2007; Russian translation: изд-во МЦНМО	

Papers	DOI or URL
A. Molev, M. Nazarov, G. Olshanski. Classical Lie algebras and Yangians. Russ. Math. Surveys (1996); Russian original: Успехи математических наук 51:2 (1996)	

Web-resources (links)	Description

Equipment
n/a

Software
n/a

Labs for Education	n/a

Knowledge

Basic results related to the universal enveloping algebras of classical Lie algebras. Basic results about the Yangians of the general linear Lie algebras and the R-matrix formalism.

Skill

Solving various problems related to the material of the course.

Experience

Experience in working with the classical Lie algebras, the R-matrix formalism, the Yangians.

Select Assignment 1 Type	Homework Assignments
Input Example(s) of Assignment 1 (preferable)	1. Compute the image of given elements in an universal enveloping algebra under the Harish-Chandra map. 2. Compute the bracket of given elements of a Poisson algebra. 3. Describe the involutive antiautomorphisms of the algebra of matrices. 4. Check the equivalence of various definitions of the Yangian Y(gl(N)). 5. Part of the exercises will consist of completing some computations or steps of proofs omitted in the lectures.
Assessment Criteria for Assignment 1	The exercises will vary in difficulty. The total score is calculated according to the formula min(100, 200*S/N), where S denotes the total number of points obtained and N denotes the maximal possible number of points