lecture #15

Two dimensional Yang-Mills in canonical formalism and many-body systems, and lifts to higher dimensions

spherical geometry —–- 3d Chern-Simons with SU(2) gauge group

$\xrightarrow{k \longrightarrow \infty}$

two dimensional Yang-Mills theory with SU(2) gauge group

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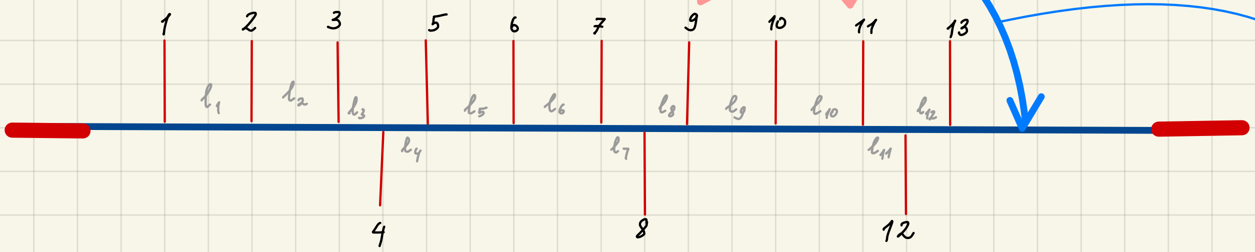

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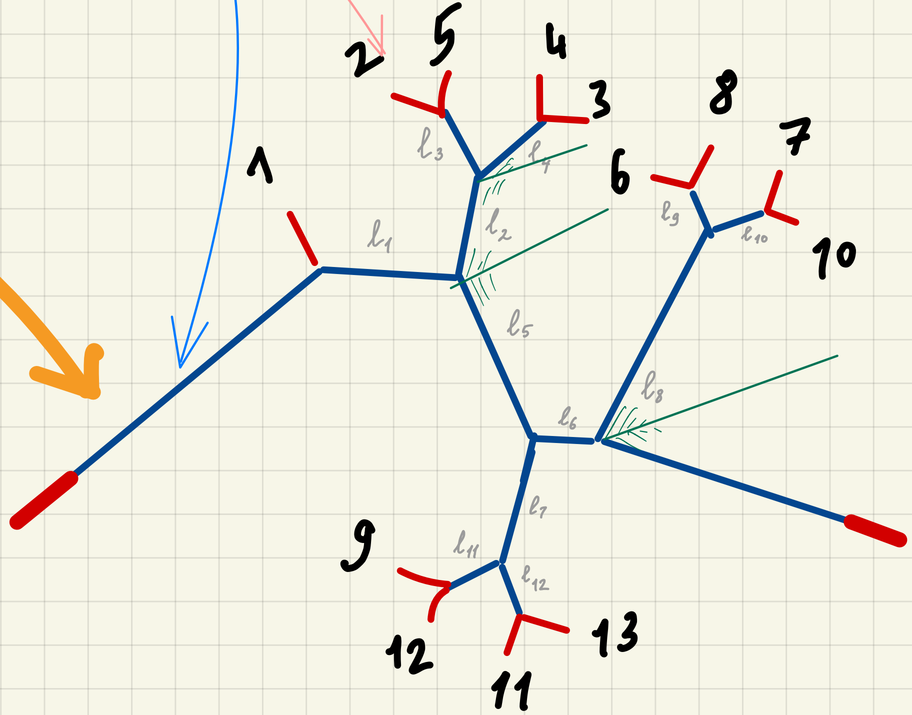
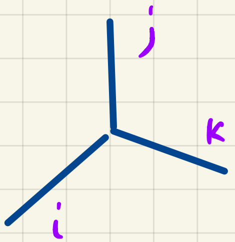
two dimensional gauge origami

$$H = \sum_i \frac{l_i}{2} \operatorname{Tr} E_i^2$$





$$E_i + E_j + E_k = 0$$

фазовое пространство теории в бесконечном объёме=

SU(2) случай: пространство замкнутых ломаных в $\mathbb{R}^3$

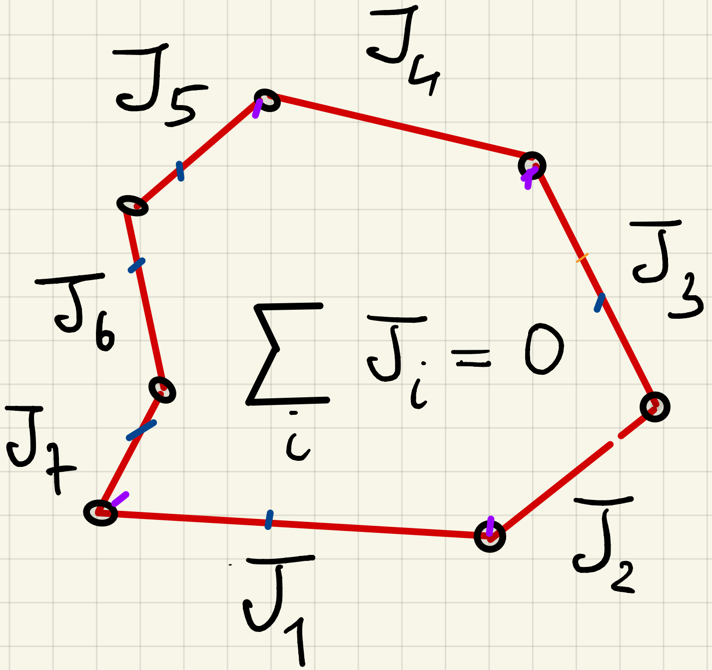
источники: $\mathbf{J}_i \in$ **Lie SU(2)**

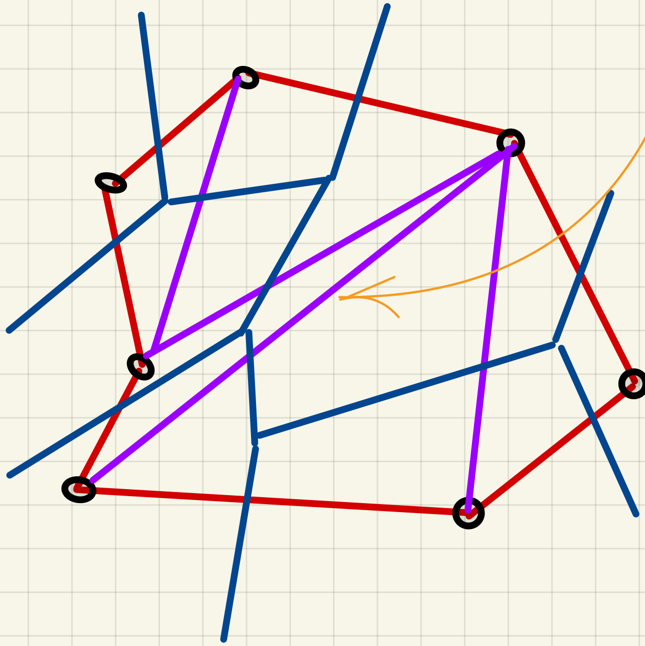
S^2

fixed eigenvalues $\mathrm{Tr}\, J_i^2 = \nu_i^2$

the flow, corresponding to this diagonal

modulo ISO(3)

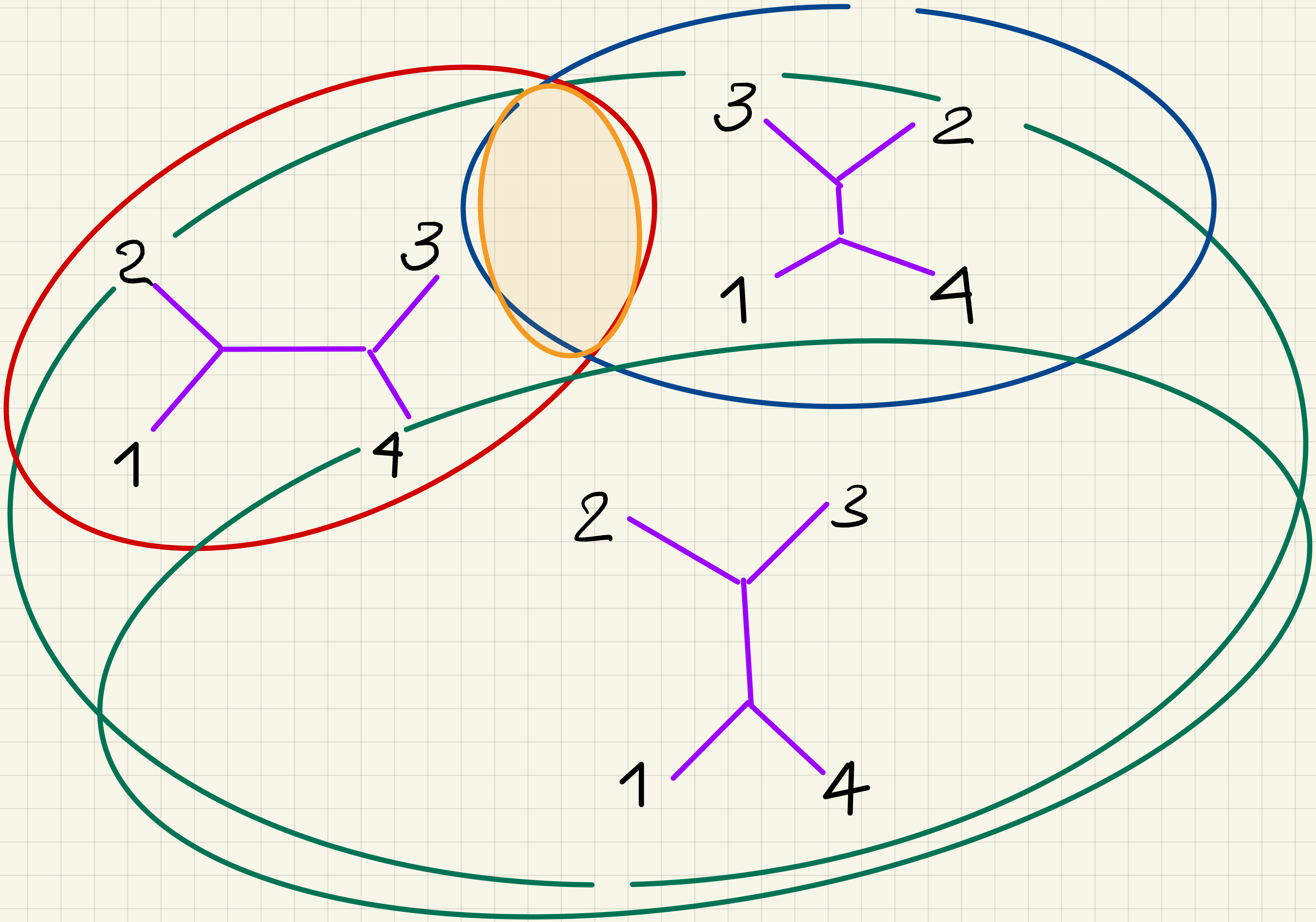




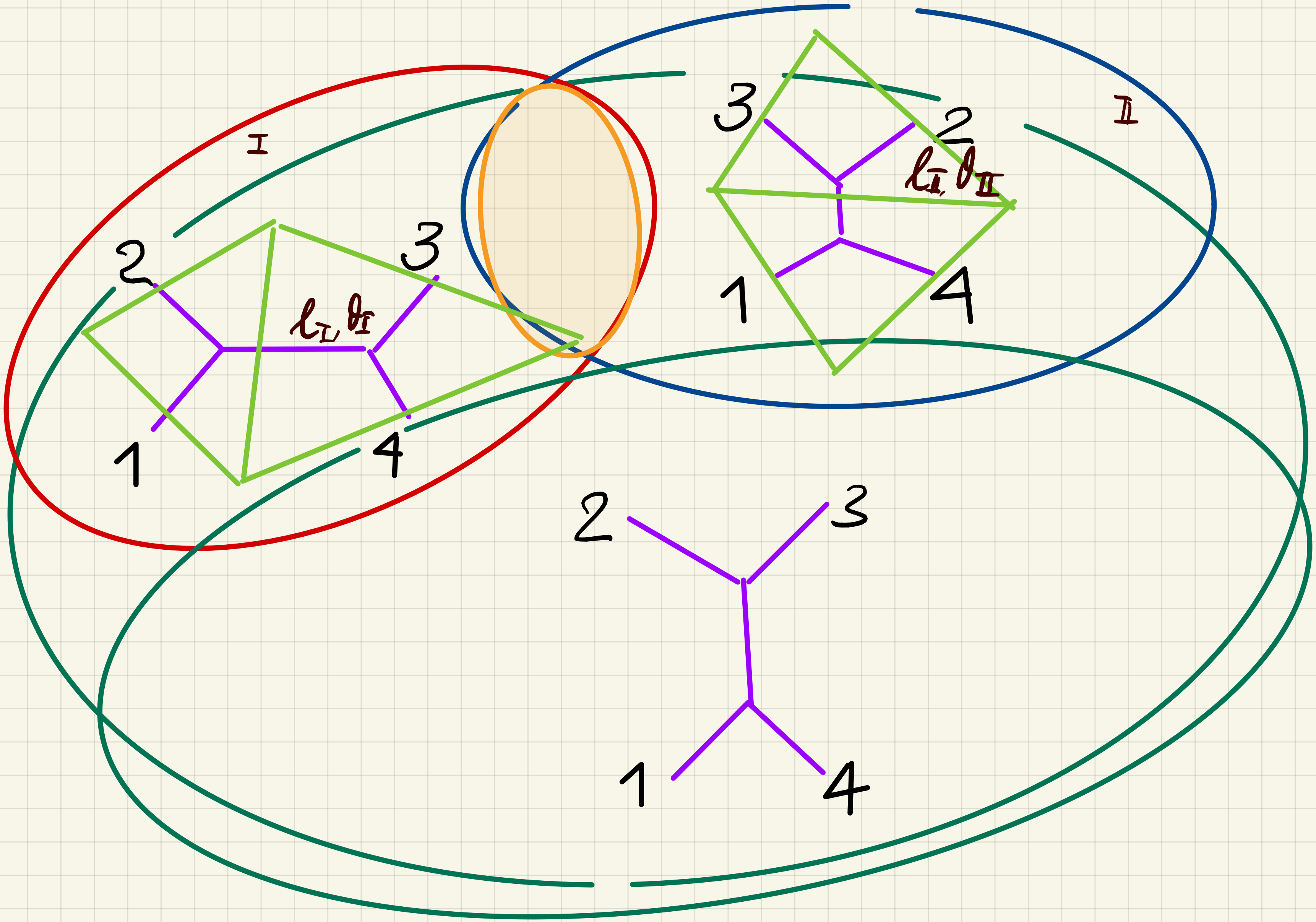
Darboux coordinates:
diagonal lengths
and dihedral angles

combinatorics of the choice of diagonals =
trivalent tree = a point on $\overline{M}_{0,N}$

triangulation= Darboux chart



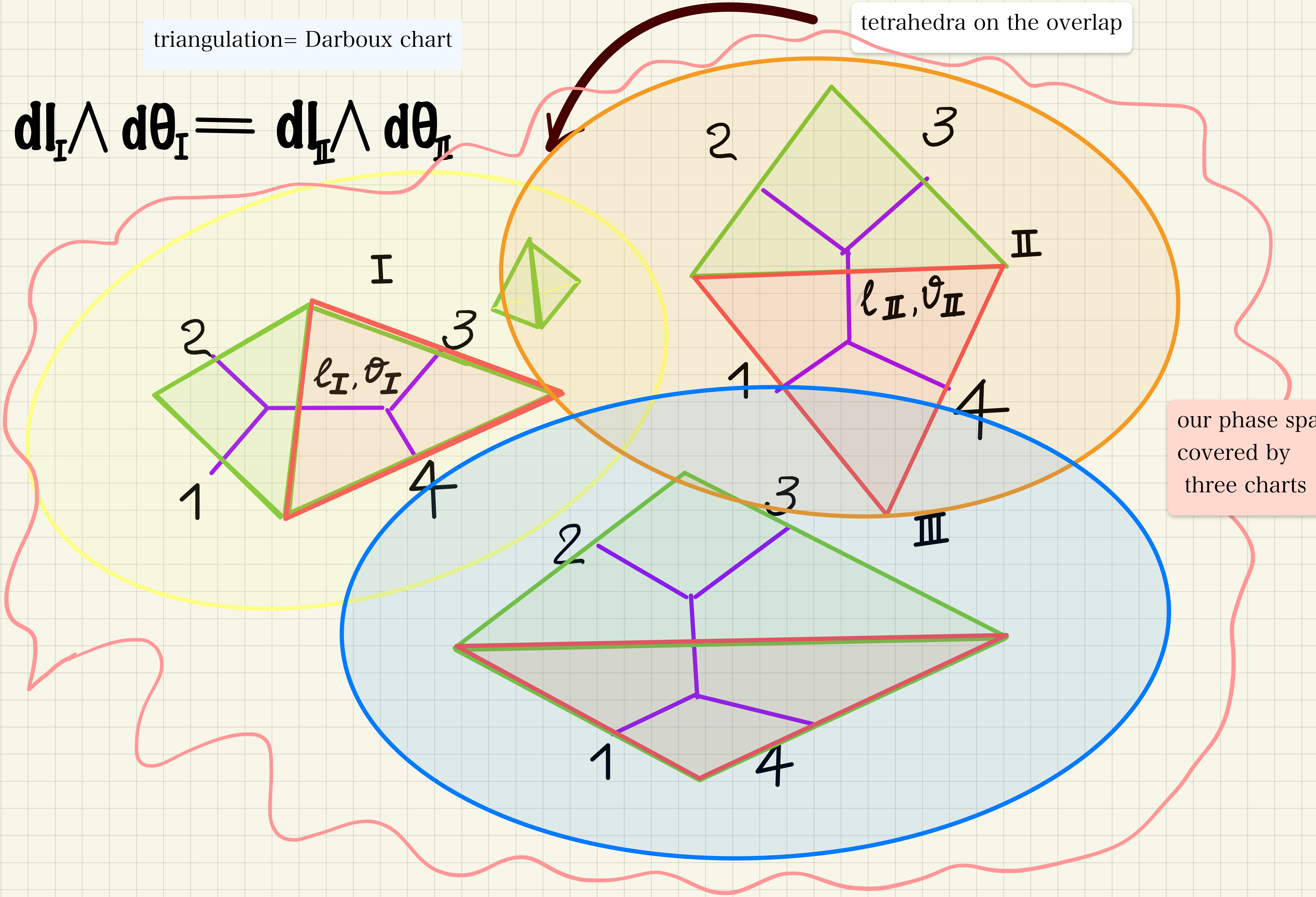
triangulation= Darboux chart



triangulation= Darboux chart

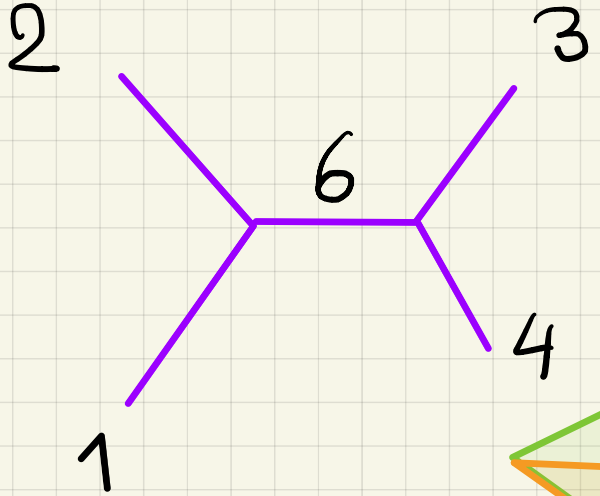
$$dl_{I} \wedge d\theta_{I} = dl_{II} \wedge d\theta_{II}$$

tetrahedra on the overlap

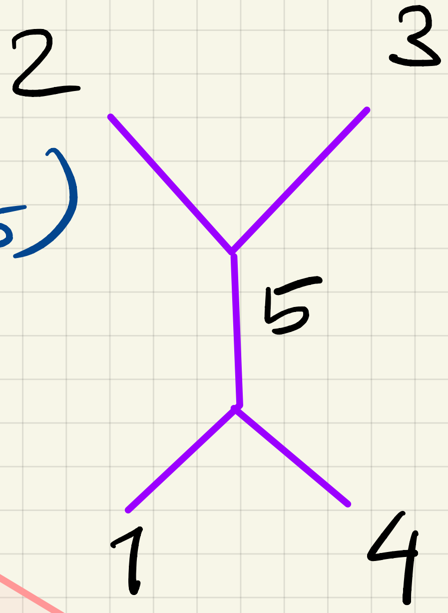


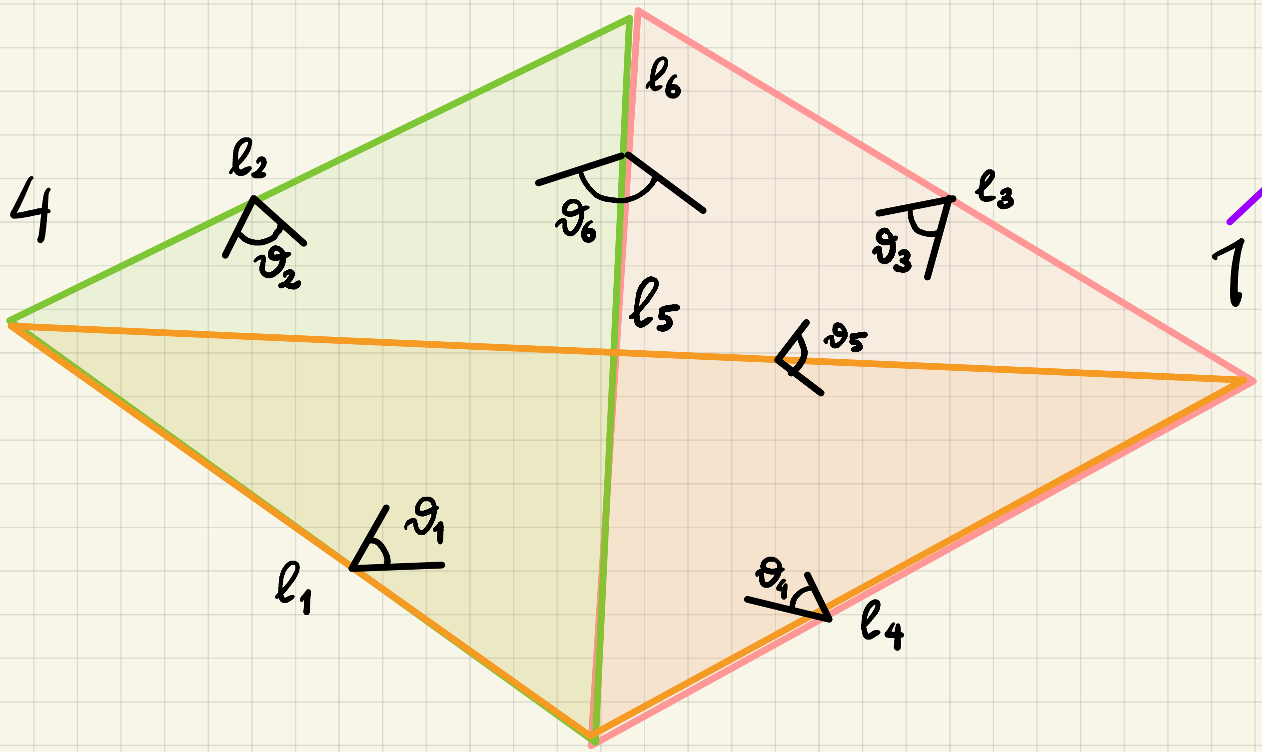
our phase space covered by three charts

on the overlap:
we see a tetrahedron



$$(\ell_6, \vartheta_6) \longmapsto (\ell_5, \vartheta_5)$$



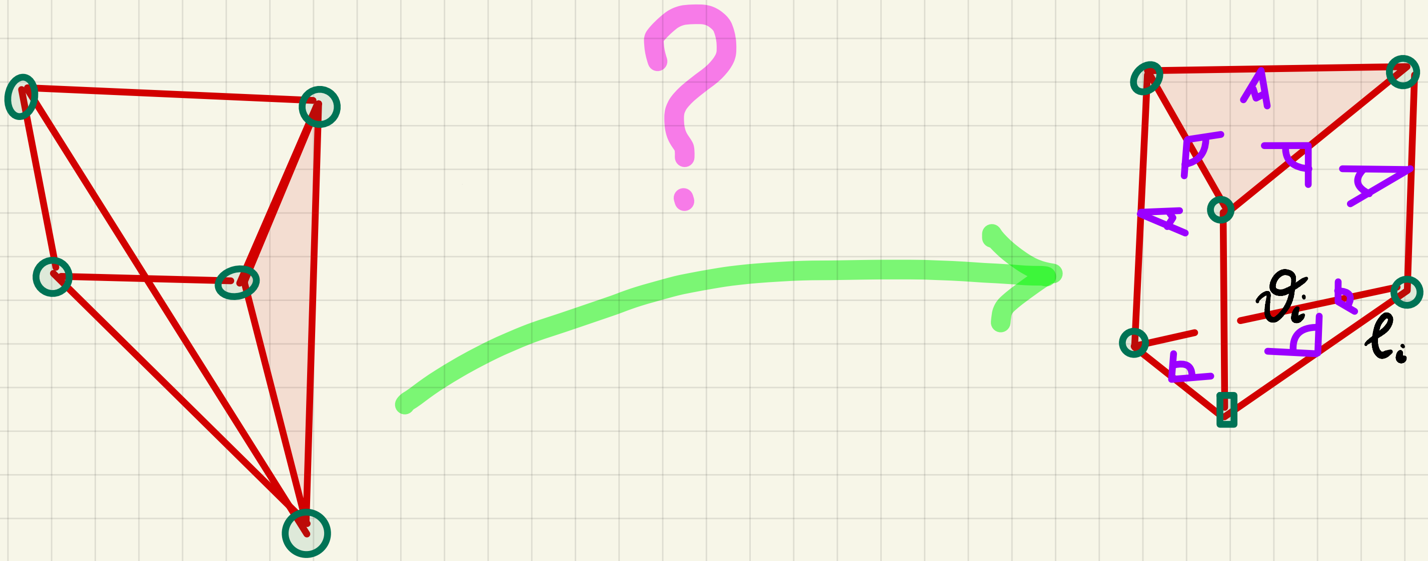


$$\sum_{i=1}^{6} \vartheta_i \, d\ell_i = dS$$

X_A linear in l_i

$$S(\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6) = \sum_A \left(X_A \log X_A - X_A \right)$$

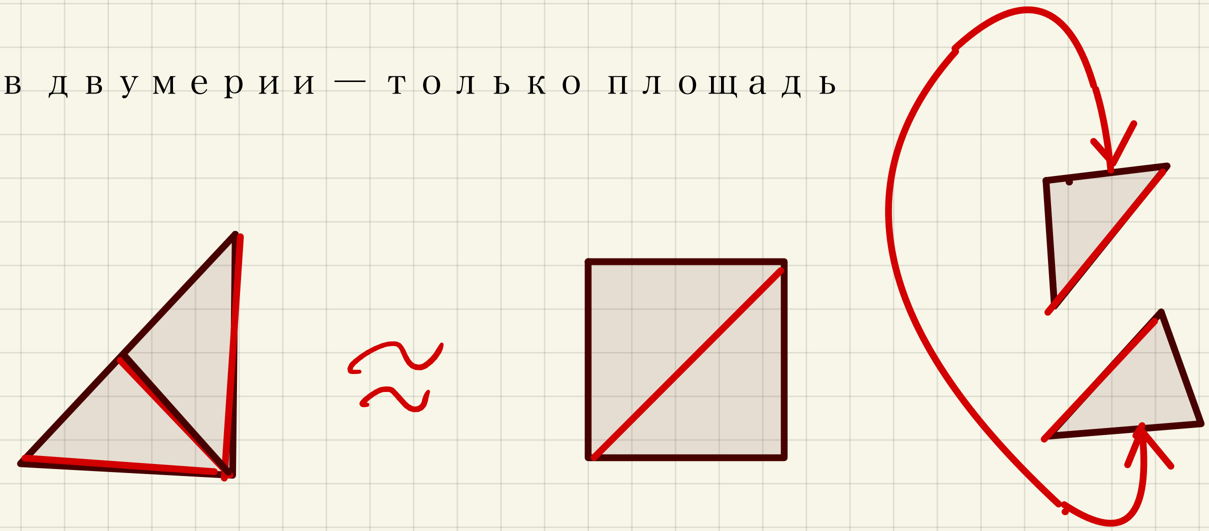
функция S связана с интересным инвариантом трехмерного многогранника (возникшего в результате решения проблемы Гильберта о конгруэнтности многогранников)



в трёхмерник, оказывается: два инварианта,
1) объём V;
2) инвариант Дена;

$$D(\Delta) = \sum_i \ell_i \otimes \vartheta_i$$

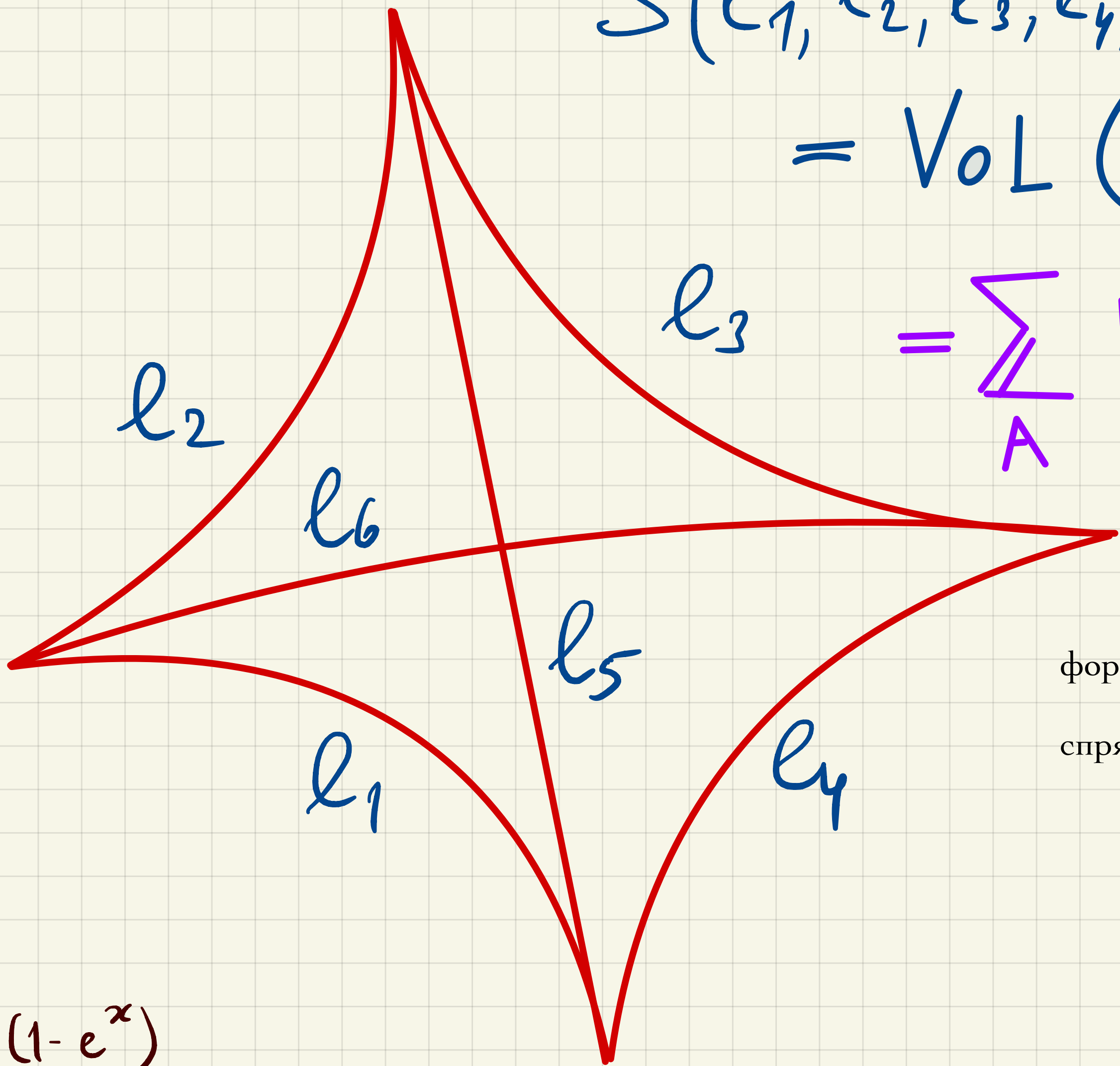
в двумерии — только площадь



$$\mathbb{R} \otimes_{\mathbb{Z}} \left(\mathbb{R} / 2\pi\mathbb{Z} \right)$$

Существует тригонометрический/гиперболический аналог

$$S(\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6) = \mathrm{Vol}(\Delta^\vee) = \sum_A \mathrm{Li}_2(e^{x_A})$$



формула Лобачевского

спрятанная Spin(8) triality

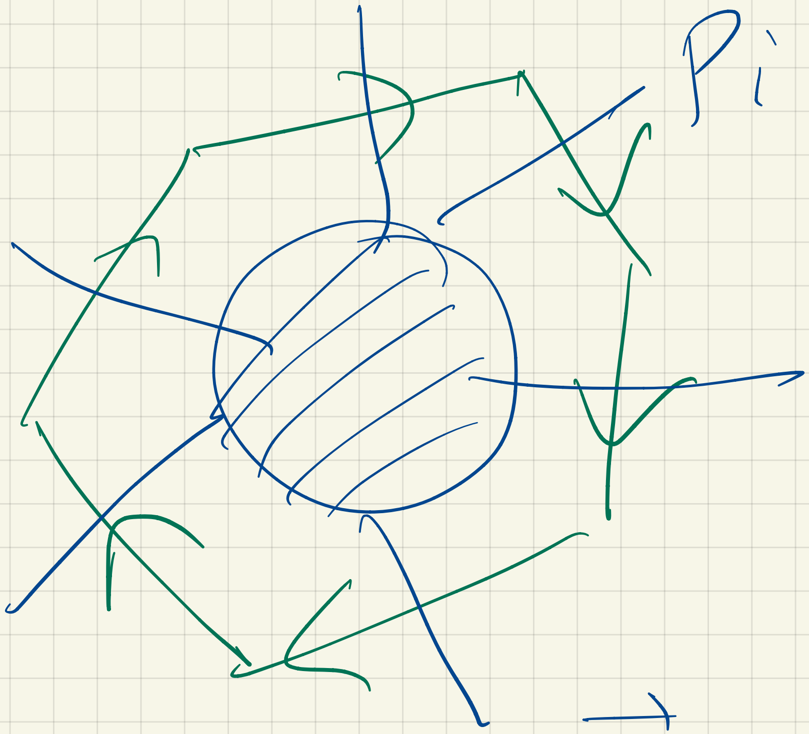
symbol of volume=Dehn invariant

H.Gangl+ NN

$$[\mathrm{Li}_2(e^x)] = e^x \otimes (1 - e^x)$$

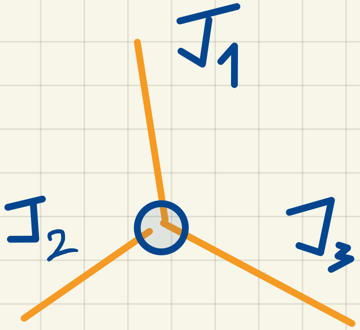
scattering amplitudes,
Wilson loops,
dual conformal invariance,
polygons in $\mathbb{R}^{1,2}$,
Kapovich-Milson coordinates

$$\vec{p}_i \in \mathbb{R}^{1,2}$$



$$\sum_i \vec{p}_i = 0$$

SU(N) case



eigenvalues $(J_i) = (\lambda_1^{(i)}, \ldots, \lambda_N^{(i)})$, $\sum_{a=1}^{N} \lambda_a^{(i)} = 0$

$$\mathbb{X}_{\lambda^{(1)},\lambda^{(2)},\lambda^{(3)}} = \left\{ (J_1, J_2, J_3) \,\middle|\, \sum_i J_i = 0,\ (J_i) \sim (g^{-1} J_i g) \right\}$$

Darboux coordinates

$\#$ positive roots - rank $= \dfrac{(N-2)(N-1)}{2}$

$$(G/T \times G/T \times G/T)/\!/G$$

$$\det\left(\lambda - \frac{J_1}{z} - \frac{J_2}{z-1}\right) = \frac{P(\mu, z)}{(z(z-1))^N}$$

$$P(\mu, z) = \det\left(\mu + zJ_3 + J_1\right)$$

$$\sum_{j=0}^{N} \mu^{N-j} \boxed{\mathrm{Tr}_{\Lambda^j \mathbb{C}^N}\left(J_1 + zJ_3\right)}$$

$$= \sum_{k=0}^{j} z^k h_k^{(j)}$$

$$\left\{ \begin{array}{l} h_0^{(j)} \\ h_j^{(j)} \\ \sum_0^j h_k^{(j)} \end{array} \right. \quad \text{det'd}$$

$$\mu = \lambda(z-1)z$$

total # of invariants = 1+2+...+(j-2)+...+(N-2)=(N-2)(N-1)/2

= A-periods of $\lambda dz = \mu \dfrac{dz}{z(z-1)}$

$$\det\left(\frac{J_1}{z} + \frac{J_2}{z-1} - \lambda\right) = \sum_{p=0}^{N} \frac{(-\lambda)^p}{(z(z-1))^{N-p}} \operatorname{Tr}_{\Lambda^{N-p}\mathbb{C}^N} \left(-J_2 - zJ_3\right)$$

$$= (-\lambda)^N \exp\left(- \sum_{i=1}^{\infty} \frac{1}{i\ \lambda^i\ z^i (z-1)^i} \operatorname{Tr}\left(J_2 + zJ_3\right)^i\right)$$

$$= (-\lambda)^N \exp\left\{ - \sum_{i=1}^{\infty} \sum_{j=0}^{i} \frac{1}{i\,\lambda^i\ z^{i-j}(z-1)^i} \sum_{1\le l_1 < \dots < l_j \le i} \operatorname{Tr} J_2^{l_1-1} J_3 J_2^{l_2-l_1-1} \cdots J_3 J_2^{i-l_j}\right.$$

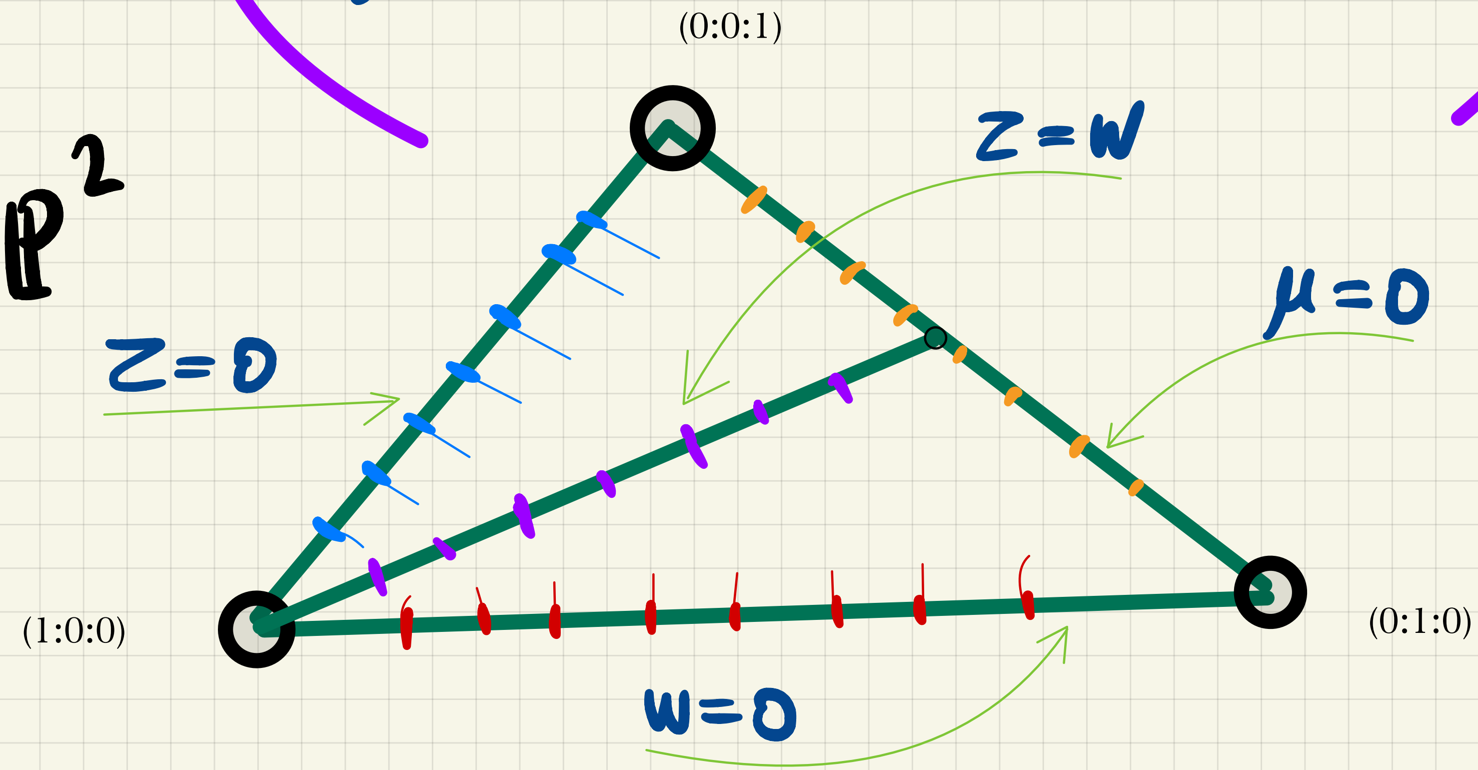
$$(l_1 - 1) + (l_2 - l_1 - 1) + \dots$$
$$+ (l_j - l_{j-1} - 1) +$$
$$+ (i - l_j) = i - j$$

curve in $\mathbb{CP}^2$

$$C_J$$

$$0 = \det\left(\mu + z J_3 + w J_1\right)$$

$$(\mu : z : w) \in \mathbb{CP}^2$$



degree N curve in $\mathbf{CP}^2$

$(\mu:z:w) \in \mathbb{CP}^2$

$\mathcal{C}_J$

$0 = \det\left(\mu + zJ_3 + wJ_1\right)$