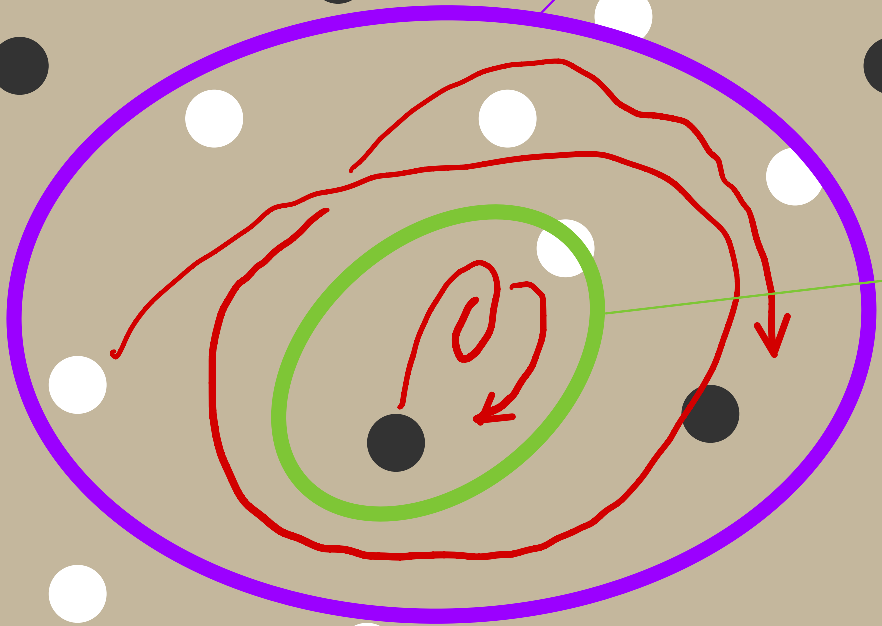
Lecture #12

фазовое пространство калибровочной теории в $d+1$ измерении



фазовое пространство системы частиц (в 1,3,4,6,9 измерениях)

toy example: finite-gap KdV solutions, Neumann flows in sigma models....

Calogero-Moser systems and gauge theories

$$H_2 = \frac{1}{2}\sum_{i=1}^{N} p_i^2 + \nu^2 \sum_{i<j} U(q_i - q_j)$$

$$P = \frac{T^*(E^N \setminus \Delta)}{S(N)}$$

$$S = \int p_i\, dq_i - H_2(p,q)\, dt$$

$$E = \mathbb{R} \to \mathbb{C},\quad \mathbb{R} \to \mathbb{C}^*,\quad S^1 \to \mathbb{C}^*,\quad S^1 \to \mathcal{E}_\tau$$

quantization

$$p_i = -i\hbar \frac{\partial}{\partial q_i}$$

$$\nu^2 \to m(m+\hbar)$$

U может зависеть от времени.....

$$U \sim \frac{1}{q^2}$$

$$U \sim \frac{1}{\sin^2(q)} \qquad \frac{1}{\sinh^2(q)}$$

$$\wp(q|\tau)$$

monopole scattering by Atiyah-Hitchin

geodesic flow on M_2^0

$$\frac{1}{2}\int g_{mn}\,\dot{x}^m \dot{x}^n$$

A.Sen $\quad H^*_{L^2}(M) \quad$ S duality in N=4 super-Yang-Mills

N.Manton's idea

$$D_\mu F_{\mu\nu} + J(\varphi) = 0 \qquad D_\mu^2 \varphi + \dots = 0$$

$$\int \mathrm{Tr}\, F_{\mu\nu}^2 + D_\mu\varphi D_\mu\varphi + V(|\varphi|)$$

+ fermion

$\varepsilon \to 0$

$t \to -\infty$

$(V \to 0)$

монополи Богомольного

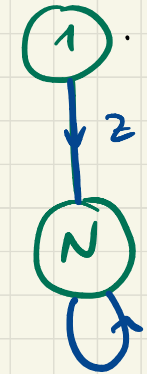
$$\vec{D}_A \varphi + \vec{B} = 0 \leftarrow 3d$$

rational Caligero-Moser

$P, Q, \quad z \in \mathbb{C}^N$

$Q \in \mathrm{Herm}_N$

gauged quantum mechanics



$$\mathcal{L}_0 = \mathrm{Tr}\; P\left(\dot{Q} + i[A, Q]\right) - \mathrm{Tr}\,P^2 + i\nu\left(\mathrm{Tr}A - a\right) + z^{+}\left(\dot{z} + iAz - iza\right)$$

$A \in \mathrm{Lie}\,U(N)$

$a \in \mathrm{Lie}\,U(1)$

$B = P + iQ$

$$\mu = \left([P, Q] + z \otimes z^{+} - \nu \mathbb{1},\; z^{+}z - \nu\right) \qquad G = \frac{U(N) \times U(1)}{U(1)}$$

$$\int p_\mu \, dq^\mu + \int \langle \mu(p,q), A_t \rangle$$

$Z_G = U(1)$

$\rightarrow A_t \qquad \mu : P \rightarrow g^*$

$$e^{\int \mathcal{L}_0 dt} = e^{\int p\,dq - \langle \mu, a \rangle} \left(e^{-i\int a_t dt} \det \left(P\exp i \int A_t dt \right) \right)^{\nu}$$

reduction with respect to **U(N)xU(1)** **U(1)**

1d Chern-Simons term

in 2n+1 dimensions

$$\int \mathrm{Tr}\, A (dA)^n + \dots$$

in two steps

step 1) **U(1)** (integrate out a)

$z^+ z = \nu$ modulo $z \mapsto e^{i\varphi} z$

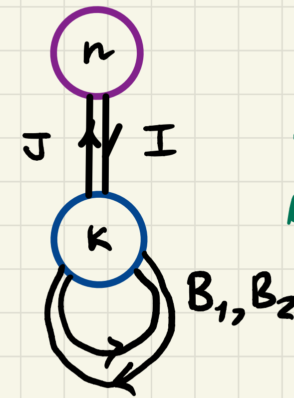
$[z] \in \mathbf{CP}^{N-1}$

step 2) $(T^*u(N) \times \mathbf{CP}^{N-1})//\mathbf{SU(N)}$

complexification

$$P, Q \to B_1, B_2$$

$$z, z^\dagger \to I, J$$



$n=1$

$k=N$

complex magnetic field

$$\mathcal{L}_0 = \mathrm{Tr}_{\mathbb{C}^k} B_1 \left(\dot{B}_2 + [A, B_2] \right) + \mathrm{Tr}_{\mathbb{C}^n} J \left(\dot{I} + A I - I a \right)$$

$$\frac{U(k) \times U(n)}{U(1)}$$

$$\mathcal{L} = \mathcal{L}_0 - \mathrm{Tr}\, B_1^2 - \zeta_{\mathbb{C}} \left(\mathrm{Tr}\, A - \mathrm{Tr}\, a \right)$$

СМС

$B_2 \longrightarrow \mathrm{diag}(q_1, q_2, \ldots, q_N)$

столкновения частиц

Однако, теперь (после комплексификации), B_2 может стать жордановой клеткой

$\mathcal{L}$ is actually an approximation in the supersymmetric quantum mechanics, where $\widetilde{\mathcal{M}}_k^{\text{framed}}(n)_5$ = configuration space

$$S = \int \mathrm{Tr}\, \dot{B}_1 \dot{B}_1^\dagger + \mathrm{Tr}\, \dot{B}_2 \dot{B}_2^\dagger + \cdots$$

$$+ \text{fermions}$$

$$= \mathcal{L} + \delta(\cdots)$$

topological supercharge

$\tilde{\mathcal{M}}_k(n)_s$ framed

$S \longrightarrow Z$

réduction/projection onto the lowest Landau level

$P = T^*\mathcal{M}$ ← фазовое

$\mathcal{M}, \omega$ – гипер кэлерово

$(z, \bar z) \in \mathcal{M}$

$(p, \bar p) \in T^*_{z,\bar z}$

$\mathcal{L}_0 = g_{i\bar j}\, \dot z^i \dot{\bar z}^{\bar j} \Longleftarrow \left(\bar p \dot z + p \dot{\bar z} - g^{-1} p \bar p \right)$

e^{iS} ↔ $k \in \mathbb{Z}$, не зависит от ℓℓℓ

$[\omega] \in H^2(\mathcal{M}, \mathbb{Z})$

$S = \int g_{i\bar j}\, \dot z^i \dot{\bar z}^j \, dt + \bar\psi\psi$

$k \int d\tau$

$+ k \int_\gamma A_i \dot z^i dt + \bar A_{\bar i} \dot{\bar z}^i dt$

$L = \mathbb{C} \times_{U(1)} P$

$\downarrow$

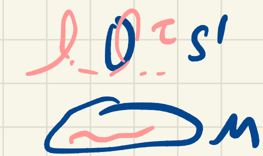
$\mathcal{M}$

$F = 2\pi i \omega = dA$ → $\omega_{\mathbb{C}}$

$U(1)$

$P \leftarrow U(1)$

$\gamma \to M$



$$\hat{H} = \nabla\bar{\nabla} + \bar{\nabla}\nabla$$ ← без фермионов

∇ – ков. произв. в L

$$\Psi \in \Gamma_{L^2}(M, L)$$

+ ферм. $\Psi \in \bigoplus_q \Omega^{0,q}_{L^2}(M, L)$

$\Psi \in H^*_{L^2}(M, L)$

$$\hat{H} = \bar{\partial}_L^+ \bar{\partial}_L \longrightarrow \bar{\partial}_L \Psi = 0$$

низшие энергии

геометрич. квантование M

Paradox

$\nu^2 U(q_i - q_j)$

$H_2 = \frac{1}{2} \mathrm{Tr} P^2 \sim \frac{1}{2} \mathrm{Tr} \frac{\partial^2}{\partial Q^2}$

ν не обязательно целое

$[P, Q] + z z^+ = \nu \cdot 1_N$ $\qquad z^+ z = \nu$

квантование ↓

$$\Psi(g^{-1} Q g, g^{-1} z) = (\det g^{-1})^{\nu} \, \Psi(Q, z) \quad (1)$$

$g \in U(N)$ $\qquad g = e^{i\theta} \cdot 1_N$ $\qquad p = N\nu \in \mathbb{Z}$

$$\Psi(Q, e^{-i\theta} z) = e^{-iN\nu\theta} \, \Psi(Q, z) \quad (2)$$

Solution of automorphic constraint: quantum Gauss law

$$\Psi(Q, z) = \left(\det g_{Q,z}\right)^{\nu} \varphi(q_{Q,z})$$

$q_{Q,z} = \mathrm{diag}(q_1, q_2, \ldots, q_N)$

$$g_{Q,z}^{-1} Q\, g_{Q,z} = q_{Q,z} \quad \text{and} \quad g_{Q,z}^{-1} z = \begin{pmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \in \mathbb{C}^N$$

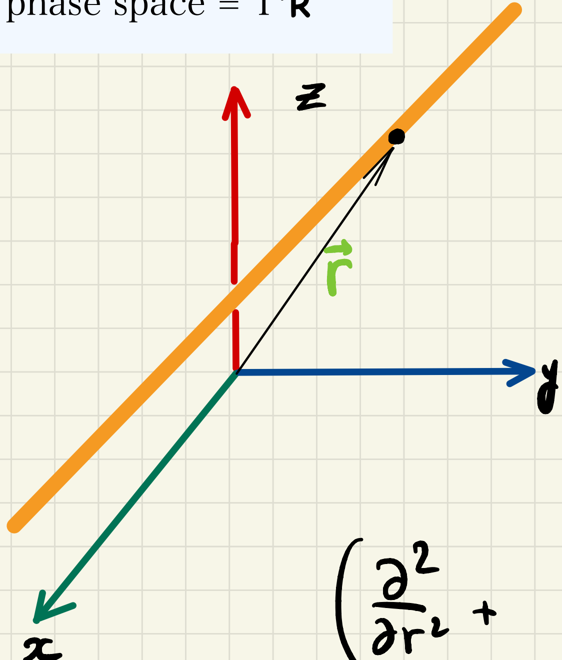
Effective Hamiltonian: Laplacian $\sum_{i,j} \frac{\partial^2}{\partial Q_{ij} \partial Q_{ji}}$ on the big space Ψ

$$\sum_{i=1}^{N} \frac{\partial^2}{\partial q_i^2} + \nu(\nu-1) \sum_{i<j} \frac{1}{(q_i - q_j)^2}$$

becomes $\rightsquigarrow h_{CM}$ on φ

geodesic motion on $\mathbb{R}^3$

phase space = $T^*\mathbb{R}^3$



2 particle case

project onto the radial direction

$$\text{Bessel}_\nu = \int_0^{2\pi} e^{i\vec{p}\cdot\vec{r}_\theta} \, e^{-i\theta\nu} \, d\theta$$

$$\vec{r}_\theta = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \vec{r}$$

$$r = |\vec{r}|$$

$$k = |\vec{p}|$$

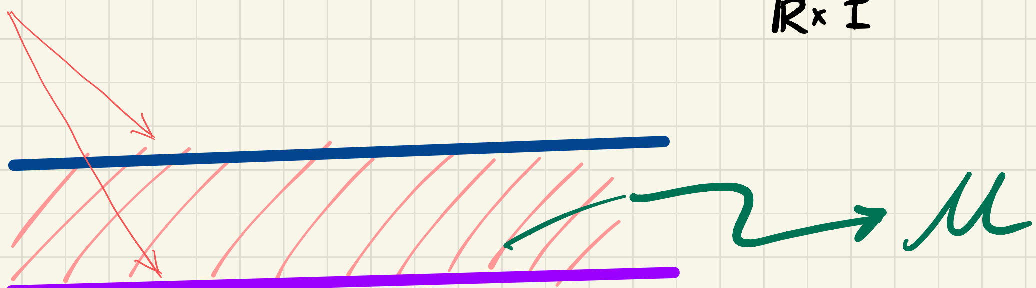
$$\left(\frac{\partial^2}{\partial r^2} + \frac{\nu^2}{r^2}\right)\Psi_\nu = k^2 \Psi_\nu$$

$$\omega \longrightarrow \omega_{\mathbb{C}}$$

$$\int_{\mathbb{R}} d^{-1}\omega_{\mathbb{C}} = \int_{\mathbb{R}\times I} \omega_{\mathbb{C}}$$

natural in the context of two dimensional N=4 sigma models

разные гранусловия



bulk action= $\delta \int d^2\sigma \; (\dots) = \int d^2\sigma \; g_{i\bar{j}}\, \partial_\mu z^{i}\, \partial_\mu \bar{z}^{\bar{j}}$ + фермионы