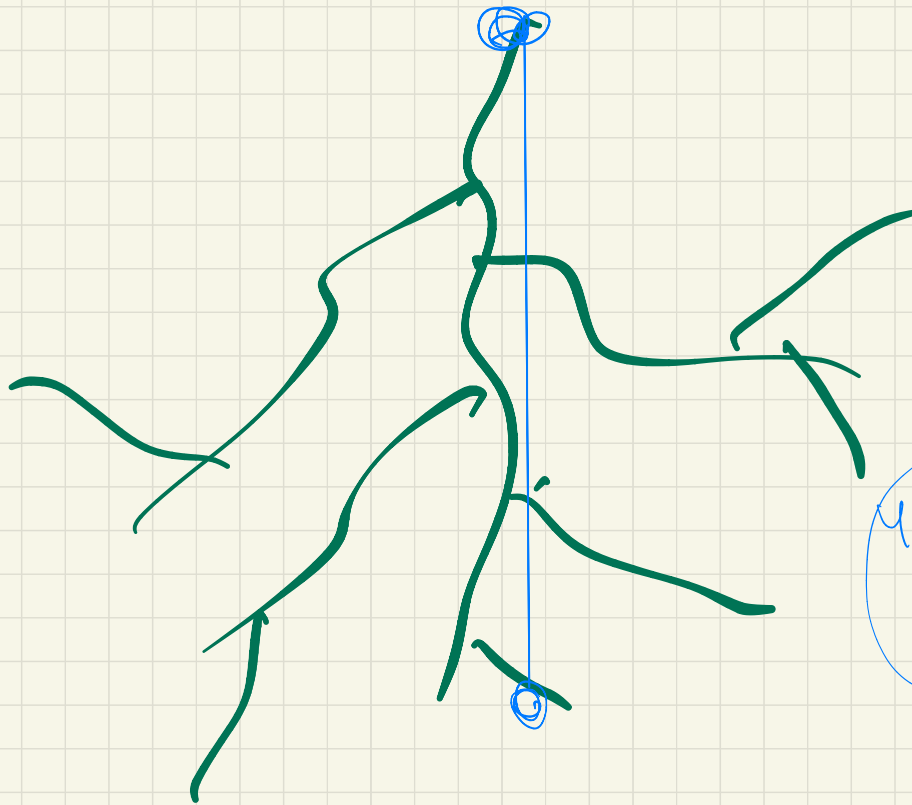
lecture #10

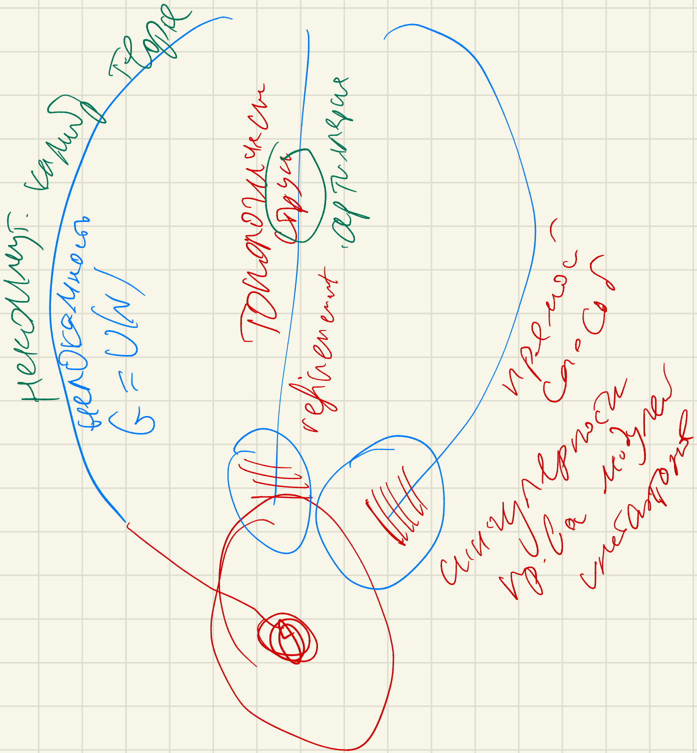
некоммутативная теория поля IV

6 матричная модель и Ω деформация

4d/2d BPS/CFT



Непертурбативни явления в суперсим. калибр. теории



$$\frac{1}{\sqrt{2\hbar}}\left(X^1 + iX^2\right) = a$$

$$\frac{1}{\sqrt{2\hbar}}\left(X^1 - iX^2\right) = a^+$$

$U(N)$ $\mathcal{N}=2$ $d=4$ SYM on $\mathbb{R}^4_\theta$ как

"матричную модель"

$Pf(\theta) \neq 0$

NxN матрица

$\to$ x некоммут.

$$X^\mu = x^\mu + i\theta^{\mu\nu} A_\nu(x)$$

$$\Phi = \phi(x)$$

$\mathcal{H}$ – представл. $\mathbb{R}^4_\theta$

"Fock"

$$\begin{cases} \Psi^i_\alpha & i=1,2 \\ & \alpha=1,2 \\ \bar{\Psi}^i_{\dot\alpha} & \end{cases}$$

X^μ, Φ – операторы в $\mathcal{H} \otimes \mathbb{C}^N$

(операторы в $\mathcal{H} \otimes \mathbb{C}^N) \otimes \mathbb{R}^{0|1}$

$$\cancel{\mathcal{S}}^{\,bos}_{\cancel{\mathcal{S}}YM} = \int \left(\mathrm{Tr}\, F_{\mu\nu}^2 + \mathrm{Tr}\, D_\mu \Phi D_\mu \bar{\Phi} + \mathrm{Tr}\,[\Phi, \bar{\Phi}]^2 \right) d^4x$$

$$\int d^4x \Updownarrow \mathrm{Pf}(\Theta)\, \mathrm{Tr}_{H} \left\{ \left([X^\mu, X^\nu] - i\theta^{\mu\nu} \right)^2 + [X^\mu, \Phi][X^\mu, \Phi^\dagger] + [\Phi, \Phi^\dagger]^2 \right\}$$

$H \cong \mathrm{Fock}_2 \otimes \mathbb{C}^N$

$x^\mu \to X^\mu$

6 matrices

$$S = \sum_{1 \le A < B \le 6} \mathrm{Tr}_H\, [X^A, X^B]^2$$

с точностью до «граничных членов»

$[A, B]$

$$\int e^{-\alpha x^2} d^4x = \left(\frac{\pi}{\alpha}\right)^2 \stackrel{?}{=} c \cdot \mathrm{Tr}_{\mathcal{H}}\, e^{-\alpha x^2}$$

$\theta_1, \theta_2 > 0$

$$e^{-\alpha x^2} \qquad [x^\mu, x^\nu] = -i\theta^{\mu\nu}$$

$$x^2 = q_1 , \quad x^1 = -i\theta_1 \frac{\partial}{\partial q_1} \longrightarrow [x^1, x^2] = -i\theta_1 \qquad [x^1, x^3] = 0$$

$$[x^3, x^4] = -i\theta_2$$

$$a_1\left(e^{-\frac{q^2}{2\theta_1}}\right) = 0$$

$$a_1 = \frac{(x^1 - i x^2)}{\sqrt{2\theta_1}} , \quad a_2 = \frac{x^3 - i x^4}{\sqrt{2\theta_2}}$$

$$(x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2 = 2\theta_1 (a_1^\dagger a_1) + 2\theta_2 (a_2^\dagger a_2) + \theta_1 + \theta_2$$

$$\mathrm{Tr}_{H}\, e^{-\alpha x^2} = e^{-\alpha(\theta_1+\theta_2)} \sum_{n_1, m_2 \geq 0} e^{-2\alpha\theta_1 n_1} e^{-2\alpha\theta_2 m_2}$$

$$= \frac{1}{\sinh(\alpha\theta_1)\sinh(\alpha\theta_2)}$$

$$\sim \frac{1}{\alpha^2 \theta_1\theta_2}$$

$\theta_1, \theta_2 \to 0$

$$\Rightarrow \quad C \sim (\pi\theta_1)(\pi\theta_2)$$

$$\int dM\, e^{-\mathrm{Tr}\, V(M)}$$

$$M \to g^{-1} M g$$

$$\int d^4x\, R^2_{\mu\nu\lambda\sigma}$$

как Diff в GR

в x-представлении невозможно определить локальную наблюдаемую

$$[\, \mathcal{O}(x), x^\mu] = i\, \theta^{\mu\nu} \frac{\partial}{\partial x^\nu} \mathcal{O}$$

инварианты

$$(X^A) \mapsto (U X^A U^\dagger) \qquad UU^\dagger = 1$$

$$\mathcal{O}_p = \int d^4x \, \mathcal{O}(x)$$

на самом деле наблюдаемых столько же

Пример

$$\mathcal{O}_p = \mathrm{Tr}_{\mathcal{H}} \, e^{i p_\mu x^\mu}$$

в вакууме, A=0

$$\mathcal{O}_p = \mathrm{Tr}_{\mathcal{H}} e^{i p_\mu x^\mu} = \frac{1}{\mathrm{Pf}(\theta)} \delta^{(4)}(p)$$

$$\mathcal{O}^{(2)}(p) = \int d^4x \, e^{ipx} \, \mathrm{Tr} F_{\mu\nu}^2(x) \quad , \ldots \int \mathrm{Tr} D_\mu F_{\alpha\beta} D_\nu F_{\alpha\beta}(x) \, d^4x \, e^{ipx}$$

$$= \mathcal{O}_{(1,2)}(p)$$

$$\mathcal{O}_{P = (p, \rho, \bar{\rho})} = \mathrm{Tr}_H \left(e^{i p_\mu X^\mu + i \rho \phi + i \bar{\rho} \bar{\phi}} \right)$$

$$\mathbb{C}^6$$

$$i \vec{p} \cdot \vec{X} \mapsto U \left(i \vec{p} \cdot \vec{X} \right) U^+$$

тензор $\mathcal{O}_{\mu\nu} = \mathrm{Tr}_H \left([X^\mu, X^\nu] \, e^{i p_\mu X^\mu + i \rho \phi + i \bar{\rho} \bar{\phi}} \right)$

в одноматричной модели

$\mathrm{Tr}\, e^{ipM} \Rightarrow \mathrm{Tr}\, \frac{1}{z-M}$

$M \qquad N \times N$

$t_i = \mathrm{Tr}\, M^i \qquad i = 1, \ldots, N$

(X^μ)

$$\phi(z) = \sum_{\mu=1}^{d} \frac{X^\mu}{z - a_\mu}$$

вспомогательные параметры, аналог импульса p_μ

$$\det_N \left(1 - \lambda^{-1} \phi(z)\right) = R(z, \lambda)$$

аналог вильсоновской линии

$\vec{X} = (x^\mu, \varphi, \bar{\varphi})$

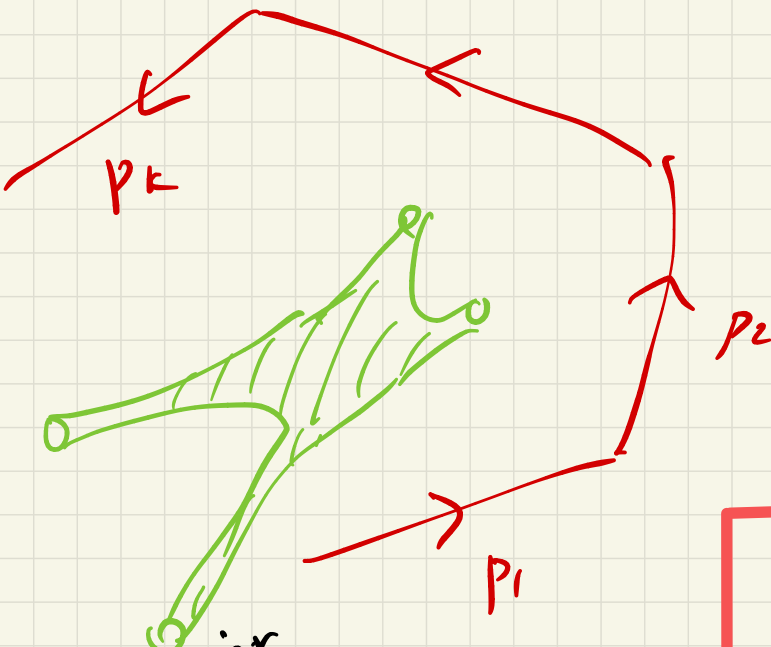
$P_i \in \mathbb{C}^6$

i=1,...,k

vertex operator
embedding 4 —-> 6

$\mathrm{Tr}_H \, e^{i \vec{p}_k \cdot \vec{X}} \cdots e^{i \vec{p}_2 \cdot \vec{X}} \, e^{i \vec{p}_1 \cdot \vec{X}}$

$R(z, \lambda)$

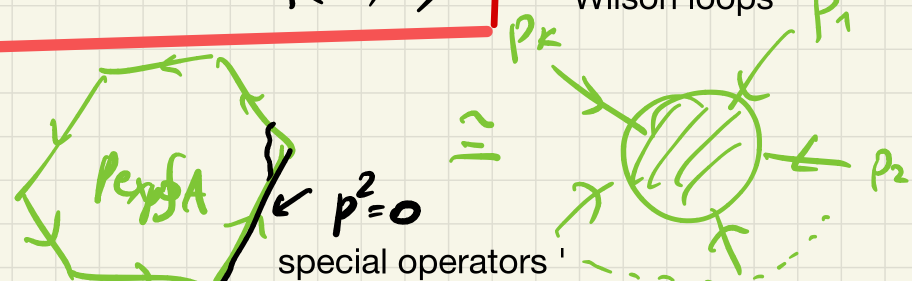


e^{ipX}

$2 \to D = 26$

N=4 super-Yang-Mills duality

scattering amplitudes
vs
Wilson loops



$$\vec{p}^{\,2} = 0$$

$$\not= \qquad \int\!\!\int \bar{p} - \bar{p}_4^2$$

$$\cap$$

$$\mathbb{C}^6$$

условия суперсимметрии

$$\delta X = \psi$$
$$\delta \psi = [X, X]$$

$$\mathrm{Tr}_H \, e^{ig\Phi}$$

Donaldson theory

$\exists$ суперзаряд

$$\delta \qquad \delta\Phi = 0$$

фермионные операторы в H

$$\begin{pmatrix} \psi_{\alpha i} \\ \bar{\psi}_{\dot\alpha i} \end{pmatrix} \longrightarrow \begin{pmatrix} \psi_\mu \\ \chi^+_{\mu\nu} \\ \eta \end{pmatrix} \begin{matrix} 4 \\ 3 \\ 1 \end{matrix}$$

$$\delta X^\mu = \psi^\mu$$

$$\delta \psi^\mu = [\varphi, X^\mu]$$

$$\delta \varphi = 0$$

$$\delta \chi^{\mu\nu} = \overbrace{[X^\mu, X^\nu] + \tfrac{1}{2}\epsilon^{\mu\nu\alpha\beta}[X^\alpha, X^\beta] - i(\theta^{\mu\nu})^\dagger \cdot 1}^{\overset{H^{\mu\nu}}{=}}$$

$$\delta \bar{\Phi} = \eta$$

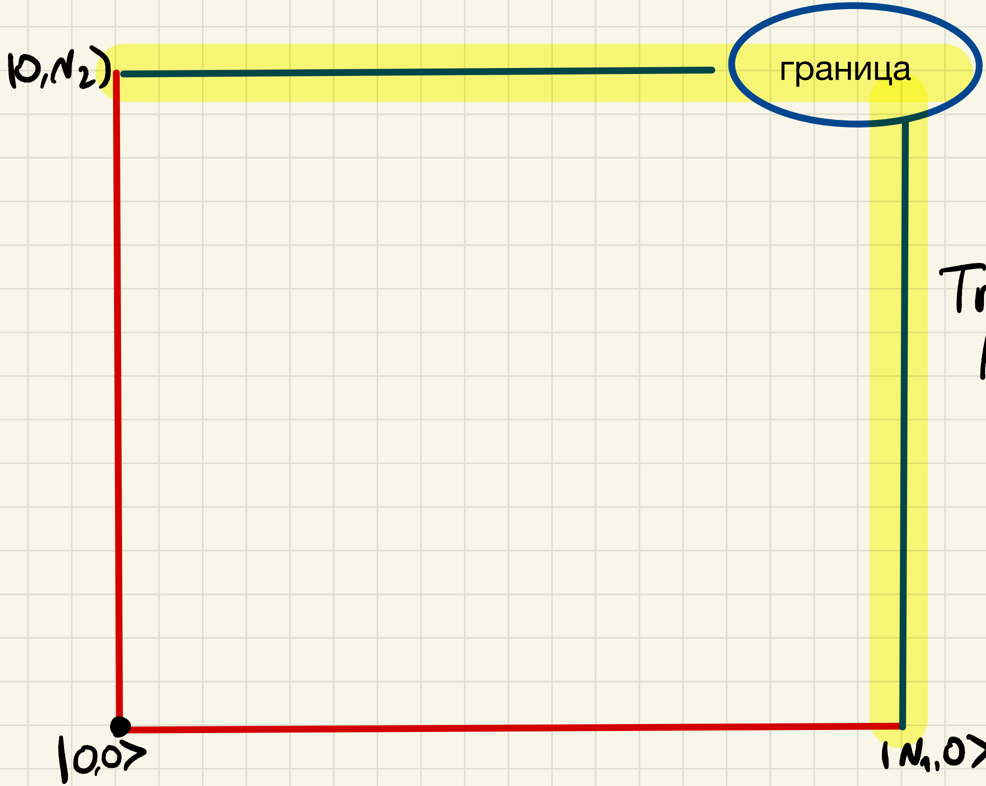
$$\delta \eta = [\varphi, \bar{\varphi}]$$

$$\delta H^{\mu\nu} = [\varphi, \chi^{\mu\nu}]$$

$$S^{nc\,YM} = \int \mathrm{Tr}_H \Big(\chi^+_{\mu\nu} \big([X^\mu, X^\nu] - i\theta^{\mu\nu} - g^2 H^{\mu\nu} \big) + \psi^\mu [X^\mu, \bar\Phi] + \eta [\Phi, \bar\Phi] \Big)$$

$$+ \tau \, \mathrm{Tr}_H \Big([X^\mu, X^\nu] - i\theta^{\mu\nu} \Big) \Big([X^\alpha, X^\beta] - i\theta^{\alpha\beta} \Big) \quad \epsilon_{\mu\nu\alpha\beta}$$

аналог инстантонного заряда

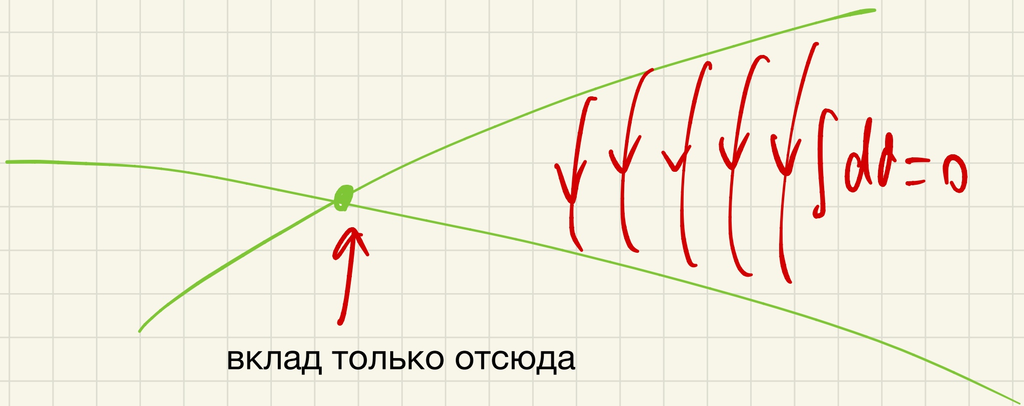


$$\mathrm{Tr}_{H_4^\infty} X^1[X^2,X^3]$$

Chern-Simons

$\delta\omega = 0$

$$\int \omega = \sum_{\text{неподвижные точки } \delta}$$



$\tilde{\mathcal{M}}_k(n) =$ пространство модулей решений

$$[X^\mu, X^\nu] + \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}[X^\alpha, X^\beta] - i(\theta^{\mu\nu})^+ \cdot 1 = 0$$

$L \cong \mathbb{R}^4$ трансляции $\curvearrowright$ $\mathbb{R}^4_\theta$

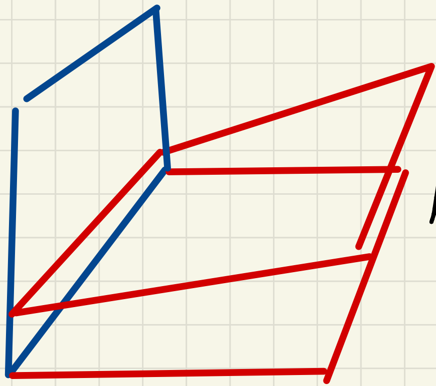
$U(2)$ вращения

Классический вакуум

$$X^\mu = \hat{x}^\mu \otimes 1_{\mathbb{C}^n}$$

$$\Phi = 1_{Fock_2} \otimes \mathrm{diag}(a_1, \dots, a_n)$$

$$S \xrightarrow{\text{кл. ур.}} 0$$



$\mathbb{R}^6$

деформируем теорию, используя вращения

$U(1)\times U(1) \curvearrowright \mathbb{R}^4_\theta$

$a_i \mapsto \exp(\sqrt{-1}\,\phi_i)\, a_i$

$a_i^+ \mapsto \exp(-\sqrt{-1}\,\phi_i)\, a_i^+$

Черн-Саймонс

деформация

$$\mathrm{Tr}_H \Big(([X^\mu, X^\nu] - i\theta^{\mu\nu})^2 + ([\Phi, X^\mu] + \Omega^\mu{}_\nu X^\nu)([\bar\Phi, X^\mu] + \bar\Omega^\mu{}_\alpha X^\alpha)$$
$$+ \big([\Phi,\bar\Phi] + \Omega^\mu{}_\nu [X^\nu, [X^\mu, \bar\Phi]] - \bar\Omega^\mu{}_\nu [X^\nu, [X^\mu, \Phi]] +$$
$$+ \Omega^\mu{}_\nu \bar\Omega^\mu{}_\alpha [X^\nu, X^\alpha]\big)^2$$

$$\mathrm{Tr}\left([\Phi,\bar\Phi] + \iota_v D_A\bar\Phi - \iota_{\bar v} D_A \Phi + \iota_v \iota_{\bar v} F_A\right)^2$$

в коммутативном случае

$$v = \Omega^\mu{}_\nu X^\nu \frac{\partial}{\partial x^\mu}, \quad \bar v = \bar\Omega^\mu{}_\nu X^\nu \frac{\partial}{\partial x^\mu}$$

Новый вакуум

$$X^\mu = \hat{x}^\mu \otimes 1_{\mathbb{C}^n} \qquad 0 = [\Phi, X^\mu] + \Omega^\mu{}_\nu X^\nu$$

$$\Phi = 1_{\mathrm{Fock}_2} \otimes \mathrm{diag}\,(a_1, \dots a_n) + \frac{1}{2} \underbrace{h_{\mu\nu} \hat{x}^\mu \hat{x}^\nu}_{\hat{h}} \otimes 1_{\mathbb{C}^n}$$

$$\hat{h} = \varepsilon_1 a_1^+ a_1 + \varepsilon_2 a_2^+ a_2$$

$$a_i \longmapsto e^{-i\hat{h}} a_i e^{i\hat{h}}$$

$$a_i^+ \longmapsto e^{-i\hat{h}} a_i^+ e^{i\hat{h}}$$

в новом вакууме

$$\mathrm{Tr}_{H}\, e^{i\rho\phi} = \frac{\sum_{i=1}^{n} e^{i\rho a_i}}{\left(1-e^{i\rho\varepsilon_1}\right)\left(1-e^{i\rho\varepsilon_2}\right)}$$