Skoltech
Skolkovo Institute of Science and Technology

Course Syllabus

Course Title (in English)	Introduction to Schramm-Loewner Evolution
Course Title (in Russian)	Введение в теорию Шрамма-Левнера
Lead Instructor(s)	Dmitry Belyaev

Contact Person	Dmitry Belyaev
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Course Description

The Schramm-Loewner Evolution (SLE) was introduced in 1998 in order to describe all possible conformally invariant scaling limits that appear in many lattice models of statistical physics. Since then the subject has received a lot of attention and developed into a thriving area of research in its own right which has a lot of interesting connections with other areas of mathematics and physics. Beyond the aforementioned lattice models it is now related to many other areas including the theory of `loop soups', the Gaussian Free Field, and Liouville Quantum Gravity. The emphasis of the course will be on the basic properties of SLE and how SLE can be used to prove the existence of a conformally invariant scaling limit for lattice models.

Topics will include:
Introduction to the theory of conformal maps, their boundary behaviour, half-plane capacity, Beurling estimates;
Loewner differential equation and its basic properties;
Introduction to stochastic analysis, Ito calculus, optional stopping theorem.
Schramm-Loewner Evolution, conformal invariance, domain Markov property, motivation, Schramm's principle;
Basic properties of SLE, phase transition, locality of SLE(6), restriction property of SLE(8/3);
Dimension of SLE curves;
Convergence to SLE, general approach, Cardy's formula.

Course Prerequisites / Recommendations	Analysis: Basic knowledge of differential equations, basic complex analysis. Probability: basic knowledge of probability Some knowledge of statistical physics (percolation, Ising model etc) will be beneficial but not required. Students will benefit from taking in parallel "Statistical mechanics, percolation theory and conformal invariance"
Аннотация	

Уравнение Шрама-Левнера играет очень важную роль в современной теории вероятности и математической физики. Оно описывает все возможные конформно-инвариантные пределы интерфейсов, которые возникают в критических моделях статистической физики. В начале этого курса я расскажу необходимые элементы теории конформных отображений, включая классическое уравнение Левнера, и стохастического анализа (в основном нам нужна формула Ито). После этого мы изучим основные свойства кривых Шрамма-Левнера и докажем, что границы между кластерами в критической перколяции сходятся к SLE(6) (теорема Смирнова).

Course Academic Level	Master-level course suitable for PhD students
Number of ECTS credits	6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Conformal maps	Half-plane capacity, mapping-out functions, Beurling estimates, Loewner Evolution	2	6	
Stochastic analysis	Ito diffusion, Ito formula, martingales, optional stopping theorem	1	3	
Schramm-Loewner Evolution	Definition, basic properties, phase transition, geometry	6	18	
Convergence to SLE	percolation, Cardy's formula, convergence of percolation to SLE(6)	2	6	

Assignment Type	Assignment Summary
Final Exam	A take-home report on one of the given topics followed by oral discussion.

Type of Assessment	Pass/Fail

Grade Structure

Activity Type	Activity weight, %
Final Exam	100

Pass: 60

Attendance Requirements Mandatory with Exceptions

Course Stream Science, Technology and Engineering (STE)

Course Term (in context of Academic Year) Term 1B (last four weeks)
Term 2

Course Delivery Frequency Only once

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Mathematical and Theoretical Physics	Mathematics and Mechanics

Course Tags Math

Required Textbooks	ISBN-13 (or ISBN-10)
Lawler, Gregory F. Conformally invariant processes in the plane. No. 114. American Mathematical Soc., 2008.	9780821846247

Recommended Textbooks	ISBN-13 (or ISBN-10)
Antti Kemppainen;Schramm-Loewner Evolution, Springer, Cham, 2017.	9783319653273
Dmitry Beliaev, Conformal maps and geometry, World Scientific 2019	9781786346131

Knowledge
Loewner and Schramm-Loewner Evolutions, their basic properties, connection between lattice models and SLE

Skill
Ability to read and understand SLE literature.

Experience
Experience in combining ideas and methods from different areas of mathematics (complex analysis, differential equations, stochastic analysis, statistical mechanics).

Select Assignment 1 Type	Final Exam
Input Example(s) of Assignment 1 (preferable)	Write an essay between 10 and 15 pages outlining the proof of the theorem about a.s. dimension of SLE(6) curves.
Assessment Criteria for Assignment 1	Pass - Meaningful discussion of the topic that shows an understanding of the main result, ideas and techniques behind the proof. Minor technicalities could be omitted. Fail - otherwise.