Skoltech
Skolkovo Institute of Science and Technology

Course Syllabus

Course Title (in English)	Statistical mechanics, percolation theory and conformal invariance
Course Title (in Russian)	Статистическая физика, просачивание и конформная инвариантность
Lead Instructor(s)	Shlosman, Semen

Contact Person	Senya Shlosman
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Course Description

This is a course on rigorous results in statistical mechanics, random fields and percolation theory. Some of it will be dedicated to the theory of phase transitions, uniqueness or non-uniqueness of the lattice Gibbs fields. We will also study the models at the criticality, where one hopes to find (in dimension 2) the onset of conformal invariance. We will see that it is indeed the case for the percolation and the Ising model.
The topics will include:
Crossing probabilities as a characteristic of sub-, super- and at- criticality.
Critical percolation and its power-law behavior.
The Russo-Seymour-Welsh theory of crossing probabilities - a cornerstone of critical percolation
Cardy's formula for crossing probabilities
Parafermionic observables and S. Smirnov theory
Conformal invariance of two-dimensional percolation a la Khristoforov.
Conformal invariance of two-dimensional Ising model
O(N)-symmetric models
Continuous symmetry in 2D systems: The Mermin–Wagner Theorem and the absence of Goldstone bosons.
The Berezinskii–Kosterlitz–Thouless transition
Reflection Positivity and the chessboard estimates in statistical mechanics
Infrared bounds and breaking of continuous symmetry in 3D

Course Prerequisites / Recommendations	This course is a continuation of my course on Statistical Physics, so some familiarity with percolation theory and the Ising model is assumed. It is desirable that the students are familiar with the probability theory, measure theory, elements of functional analysis, and complex analysis. Of course, calculus knowledge is assumed.

Аннотация

Мой курс посвящён строгим результатам статистической механики и теории просачивания. Я расскажу о некоторых элементах теории фазовых переходов в решётчатых моделях (типа модели Изинга). Значительная часть курса будет посвящена конформной инвариантности двумерной теории просачивания в критической точке и аналогичного утверждения для модели Изинга.
Я расскажу о поведении вероятностей наличия связи между противоположными сторонами прямоугольников (crossing probabilities) в зависимости от значения вероятности просачивания. Это поведение различает три режима: критический, надкритический и подкритический.
Центральной частью курса является изучение двумерных моделей в критическом режиме. Я расскажу очень красивое доказательство теоремы Смирнова о конформной инвариантности двумерного просачивания, найденное М. Христофоровым.
Слушатель курса научится читать и понимать статьи о конформной инвариантности в статистической физике. Он овладеет методами строгого изучения явления фазового перехода.

Course Academic Level	Master-level
Number of ECTS credits	6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Basic notions of percolation theory. Crossing probabilities.	The behavior of the probabilities of having a left-right crossing in the rectangle box. Subcritical, supercritical and critical case.	1	3	
The critical value of the percolation parameter. Kesten theorem: p_c=1/2.	The value of the critical probability. Sharpness of the criticality. Power-law decay of connectivities.	1	3	
The main technique of the critical percolation: the Russo-Seymour-Welsh theory	Crossing probabilities for rectangles with arbitrary aspect ratio: recent versions and proofs.	1	3	
Cardy’s formula	The exact formula for crossing probabilities, as correctly predicted by physicists on the basis of conformal invariance conjecture.	1	3	
Parafermionic observables	The exact meaning of conformal invarinace. The key ingredients of the proof of conformal inariance.	1	3	
Conformal invariance of two-dimensional percolation	The proof of the Theorem of Stanislav Smirnov -- the Khristoforov version.	1	3	
Conformal invariance of two-dimensional Ising model	Correlation functions and their scaling at criticality.	1	3	
O(N)-symmetric models	Systems with continuous symmetry in 2D and 3D. Open problems. Continuous symmetry breaking.	1	3	
The Mermin–Wagner Theorem	2D models with continuous symmetry. No Goldstone bosons in 2D. Is continuous symmetry breaking in 2D possible?	1	3	
The Berezinskii–Kosterlitz–Thouless transition	Vortices and the Coulomb gase in 2D. Dipoles. Debye screening.	1	3	
Reflection Positivity and the chessboard estimates. Infrared bounds	Methods of the field theory in statistical mechanics. Continuous symmetry breaking in 3D.	1	3	

Assignment Type	Assignment Summary
Final Exam	This will be a home-written report on a topic selected by me, with a following oral discussion of it and of related topics.
Class participation	My habit is to ask questions during the lectures. The attendees are assumed and encouraged to ask questions themselves. The best strategy for a student is to ask many questions and especially to point (politely) to the mistakes of the professor.
Report	A home-written midterm report on a topic selected by me.

Type of Assessment: Pass/Fail

Grade Structure

Activity Type	Activity weight, %
Final Exam	40
Class participation	30
Report	30

Pass: 60

Attendance Requirements: Mandatory with Exceptions

Maximum Number of Students

	Maximum Number of Students
Overall:	20
Per Group (for seminars and labs):	20

Course Stream: Science, Technology and Engineering (STE)

Course Term (in context of Academic Year): Term 3

Course Delivery Frequency: Every two years

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Mathematical and Theoretical Physics	Mathematics and Mechanics

Course Tags: Math

Required Textbooks	ISBN-13 (or ISBN-10)
Theory of Phase Transitions: Rigorous Results by Ya. G. Sinai	9780080264691
G. Grimmett. Percolation. Springer-Verlag, Berlin, 1999.	978-3-642-08442-3

Recommended Textbooks	ISBN-13 (or ISBN-10)
Statistical Physics L D Landau E.M. Lifshitz	9780080570464

Papers	DOI or URL
Dmitry Chelkak and Stanislav Smirnov. Universality in the 2D Ising model and conformal invariance of fermionic observables. Invent. Math., 189(3):515–580, 2012.	

Web-resources (links)	Description
http://www.unige.ch/math/folks/velenik/smbook/index.html	Statistical Mechanics of Lattice Systems: a Concrete Mathematical Introduction
https://www.ihes.fr/~duminil/publi/2017percolation.pdf	Lectures on percolation theory.

Equipment
laptop

Software
Mathematica

Labs for Education	None

Knowledge
Theory of random fields, in particular, Markov fields and Gibbs fields. Mathematical theory of phase transitions. Random surfaces. Critical phenomena. Conformal invariance

Skill
Ability to read and understand the literature on rigorous statistical physics and percolation theory, e.g. Journal of Statistical Physics and (some) papers in Communications in Mathematical Physics.
Ability to formulate and sometime also solve problems in the theory of phase transitions and related areas.

Experience
Experience of working in the area of mathematical physics and probability theory

Select Assignment 1 Type	Report
Input Example(s) of Assignment 1 (preferable)	Prove the existence of phase transition for the Ising model on uniform (infinite) Cayley tree.
Assessment Criteria for Assignment 1	A - for the correct proof. For wrong or incomplete proof - depending on the sketch.
Select Assignment 2 Type	Class participation
Input Example(s) of Assignment 2 (preferable)	1. I ask a question: can one define the conformal invariance in 3D? 2. The student asks a question: is percolation continuous in 3D?
Assessment Criteria for Assignment 2	1. A - for a meaningful discussion of the topic. No assessment otherwise. 2. A - for a meaningful discussion of the topic. Fields medal for a complete solution.
Select Assignment 3 Type	Final Exam
Input Example(s) of Assignment 3 (preferable)	Please give a proof of the absence of breaking of the continuous symmetry in 2D systems.
Assessment Criteria for Assignment 3	A - for the correct proof. For wrong or incomplete proof - depending on the sketch.
Free Style Comments (if any)	I prefer students to ask as many questions as possible. I do not mind taking turns and deviate from the initial plan of my lectures.