Skoltech
Skolkovo Institute of Science and Technology

Course Syllabus

Course Title (in English)	Affine quantum groups
Course Title (in Russian)	Аффинные квантовые группы
Lead Instructor(s)	Bershtein, Mikhail

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Course Description

The main source of the theory of quantum groups was integrable models. Affine quantum groups (better to say quantum universal enveloping of affine Lie algebras) are symmetries of the XXZ model, they appeared simultaneously with quantum group theory in the mid-80s. Affine quantum groups have many new properties (compared to quantum universal enveloping algebras of simple Lie algebras) --- different realizations, different comultiplications. They have many applications, in addition to the integrable models mentioned above, we mention cluster algebras and geometric representation theory.

In the course, we will discuss the basic constructions associated with quantum affine algebras and touch on their applications. Some basic familiarity with quantum groups and affine Lie algebras is assumed, and familiarity with the Bethe ansatz is also desirable.

Course Prerequisites / Recommendations	Some basic knowledge of quantum groups is required. Some basic knowledge about affine Lie algebras is required. Some acquaintance with Bethe ansatz will be helpful.

Аннотация

Основным источником теории квантовых групп была теория интегрируемых моделей. Аффинные квантовые группы (более правильно говорить квантовые универсальные обертывающие алгебры от аффинных алгебр Ли) являются симметриями XXZ модели, они возникли одновременно с теорией квантовых групп в середине 80-х. Аффинные квантовые группы имеют много новых свойств (по сравнению квантовыми универсальными обертывающими простых алгебр Ли) --- разные реализации, разные коумножения. Они имеют много приложений, помимо упомянутых выше интегрируемых моделей, отметим кластерные алгебры и с геометрическую теорию представлений.

В рамках курсе мы обсудим основные конструкции связанные с квантовыми аффинными алгебрами и коснемся их приложений. Некоторое предварительно знакомство с квантовыми группами и аффинными алгебрами Ли предполагается, также, желательно знакомство с анзацем Бете.

Course Academic Level	PhD-level suitable for MSc students
Number of ECTS credits	6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Review of quantum groups	Example sl(2), Lie bialgebras, Drinfeld-Jimbo formulas. Lusztig's Braid group action. Universal matrix. RTT realization.	1	2	2
Affine lie algebras anf groups	Definition, Manin triples in affine settings. Yangians.	1	2	2
Evaluation representations	Left and right dual representations. Tensor products. Finite R matrices with spectral parameter. Examples in type D.	1	2	2
New Drinfeld realization	Isomorphism with standard one via braid group action	1	2	2
Bosonization.	Bosonization of basic representations/ Vertex operators for different coproducts	1	2	2
RTT realization	Isomorphism between RTT realization and new realization. Coproducts.	1	2	2
Hecke algebras	Hecke algebras. Shur-Weyl duality for quantum groups, for affine quantum groups.	1	2	2
Finite dimensional representations	Affine sl(2) case. Kirillov-Reshetikhin modules. Drinfeld polynomials. q-Characters.	1	2	2
XXZ integrable models	XXZ integrable models. R matrix definition. Algebraic Bethe ansatz.	1	2	2
TQ relations	From TQ relations to Bethe ansatz. TQ relation from representations of sl(2). Prefundametal representations.	1	2	2
q-Knizhnik–Zamolodchikov equations	Knizhnik–Zamolodchikov equations. Poles of the Hamiltonians. Commutativity. Dynamical equations. Commutativity.	1	2	2

Assignment Type	Assignment Summary
Class participation	This includes attendace and activity during the lectures and seminars
Problem Set	Each lecture will be supplied with list of problems, their solution is required.

Type of Assessment	Graded

Grade Structure

Activity Type	Activity weight, %
Class participation	50
Problem Set	50

A:	86
B:	76
C:	66
D:	56
E:	46
F:	0
Attendance Requirements	Mandatory with Exceptions
Course Stream	Science, Technology and Engineering (STE)
Course Term (in context of Academic Year)	Term 1 Term 2

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Mathematical and Theoretical Physics	Mathematics and Mechanics

Course Tags

Math

Required Textbooks	ISBN-13 (or ISBN-10)
Pavel I. Etingof, Igor Frenkel, Alexander A. Kirillov Lectures on Representation Theory and Knizhnik-Zamolodchikov Equations	9780821804964
Michio Jimbo, Tetsuji Miwa, Algebraic Analysis of Solvable Lattice Models	978-0-8218-0320-2

Recommended Textbooks	ISBN-13 (or ISBN-10)
Vyjayanthi Chari, Andrew N. Pressley A Guide to Quantum Groups	978-0521558846

Papers	DOI or URL
Jin Tai Ding and Igor B. Frenkel, "Isomorphism of two realizations of quantum affine algebra Uq(germggerml(n))"	10.1007/BF02098484
igor b. Frenkel and Naihuan Jing, "Vertex representations of quantum affine algebras"	10.1007/BF00401590
David Hernandez and Michio Jimbo, "Asymptotic representations and Drinfeld rational fractions"	0.1112/S0010437X12000267
David Hernandez, Bernard Leclerc "Quantum affine algebras and cluster algebras"	
Beck "Braid group action and quantum affine algebras"	10.1007/BF02099423

Knowledge
Basic notions on the theory of the affine quantum groups: three different realizations, Bethe ansatz and Bethe equations, Drinfeld polynomials

Skill
Computations in noncommutative algebras similar to universal enveloping to Lie algebras.

Experience
revealing algebraic structure in problems of mathematical physics.

Select Assignment 1 Type	Class participation
Assessment Criteria for Assignment 1	the score of student is proportional to the number of classes in which he participated