Skoltech
Skolkovo Institute of Science and Technology

Course Title (in English)	Research seminar "Cluster integrable systems and supersymmetric gauge theories"
Course Title (in Russian)	Научно-исследовательский семинар "Кластерные интегрируемые системы и суперсимметричные калибровочные теории"
Lead Instructor(s)	Gavrylenko, Pavlo Marshakov, Andrey
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Course Description

This research seminar will be devoted to the study of N=2 supersymmetric gauge theories
and related topics. It turns out that comparing to the N=1 theories, N=2 supersymmetry allows to compute much more quantities. In particular, low-energy effective action can be
described in terms of single function, prepotential. Seiberg-Witten solution of the N=2 theory gives explicit description of the prepotential in terms of periods of some meromorphic differential on algebraic curve. It turns out that this description is deeply related to classical integrable systems.

This will be the working seminar where we are going to discuss some topics related to Seiberg-Witten theory in 4D and 5D: partition functions, relation to the integrable systems and their deautonomizations (isomonodromic deformations). On the integrable systems side we will consider cluster integrable systems (like relativistic Toda chains), which come from the dimer models or from double Bruhat cells in Poisson-Lie groups. We are going to discuss their Lax representation, discrete and continuous flows, relation to the dimer models, etc.

We expect some talks given by participants.

Course Prerequisites / Recommendations	Algebraic curves, Seiberg-Witten exact solution, Lax representation
Аннотация	

Этот научно-исследовательский семинар будет посвящён изучению N=2 суперсимметричных калибровочных теорий и связанных сюжетов. Оказывается, что по сравнению с N=1 теориями, N=2 суперсимметрия позволяет значительно больше величин. В частности, низкоэнергетическое действие может быть выражено в терминах одной функции, препотенциала. Решение Заберга-Виттена N=2 теории даёт явное описание препотенциала в терминах периодов некоторого мероморфного дифференциала на алгебраической кривой. Оказывается, что это решение глубоко связано с классическими интегрируемыми системами.

Это будет рабочий семинар, на котором мы собираемся обсуждать некоторые темы, связанные с четырёхмерной и пятимерной теориями Зайберга-Виттена: их статсуммы, свящь с интегрируемыми системами и их деавтономизациями (изомонодромными деформациями). На стороне интегрируемых систем мы рассмотрим кластерные интегрируемые системы (типа релятивистских цепочек Тоды), которые приходят из димерных моделей или с двойных клеток Брюа в группах Пуассона-Ли. Мы собираемся рассмотреть их представления Лакса, дискретные и непрерывные потоки, связь с димерными моделями, и т.д.

От участников ожидаются доклады.

Course Academic Level	Master-level course suitable for PhD students
Number of ECTS credits	6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Basics of dimer integrable systems	Dimer models. Poisson-Lie groups and symplectic leaves inside them. Goncharov-Kenyon construction, its connection to Fock-Marshakov construction. Definition of the cluster algebra, discrete flows.	0	6	0
Zoo of cluster integrable system	Relativistic Toda chains for PGL(N) and for other series, XXZ spin chains, etc.	0	6	0
Relation between integrable systems and Seiberg-Witten theory	Spectral curves of 5D Seiberg-Witten theories as partition functions of dimer models. Deautonomization of corresponding integrable systems and their solutions in terms of Nekrasov partition functions.	0	6	0
Other topics	Some other ideas can be considered during the seminar.	0	31	0

Assignment Type	Assignment Summary
Presentation	To give a talk on some topic related to Seiberg-Witten theory or/and cluster integrable systems
Attendance	To participate in the discussions during seminars, to understand what is going on

Type of Assessment	Graded

Grade Structure		

Activity Type	Activity weight, %
Attendance	70
Presentation	30

A:	86
B:	76
C:	66
D:	56
E:	46
F:	0

Attendance Requirements	Mandatory with Exceptions

Course Stream	Science, Technology and Engineering (STE)

Course Term (in context of Academic Year)	Term 1 Term 2

Course Delivery Frequency	Every year

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Mathematical and Theoretical Physics	Mathematics and Mechanics

Course Tags	Math Physics

Required Textbooks	ISBN-13 (or ISBN-10)
Andrei Marshakov, Seiberg-Witten Theory and Integrable Systems	978-981-4495-57-8

Papers	DOI or URL
L. Alvarez-Gaume, S.F. Hassan, Introduction to S-Duality in N=2 Supersymmetric Gauge Theory. (A pedagogical review of the work of Seiberg and Witten)	https://arxiv.org/abs/hep-th/9701069
Adel Bilal, Duality in N=2 SUSY SU(2) Yang-Mills Theory: A pedagogical introduction to the work of Seiberg and Witten	https://arxiv.org/abs/hep-th/9601007
Nikita A. Nekrasov, Seiberg-Witten Prepotential From Instanton Counting	https://arxiv.org/abs/hep-th/0206161
Nikita Nekrasov, Vasily Pestun, Seiberg-Witten geometry of four dimensional N=2 quiver gauge theories	https://arxiv.org/abs/1211.2240
A. B. Goncharov, R. Kenyon, Dimers and cluster integrable systems	https://arxiv.org/abs/1107.5588
A. Marshakov, Lie Groups, Cluster Variables and Integrable Systems	https://arxiv.org/abs/1207.1869
V. V. Fock, A. Marshakov, Loop groups, Clusters, Dimers and Integrable systems	https://arxiv.org/abs/1401.1606
M. Bershtein, P. Gavrylenko, A. Marshakov, Cluster Toda chains and Nekrasov functions	https://arxiv.org/abs/1804.10145

Knowledge
Cluster integrable systems, their spectral curves, Lax representations. Exact solution of 5D Seiberg-Witten theory and its relation to cluster integrable systems.

Skill
Ability to find the spectral of the cluster integrable system, to compute its Hamiltonians, to describe what the corresponding Seiberg-Witten theory is.

Experience
To prepare talks and to participate in the discussions.

Select Assignment 1 Type	Presentation

Input Example(s) of Assignment 1 (preferable)	To give a talk on some topic related to Seiberg-Witten theory

Assessment Criteria for Assignment 1	The student expected to give a talk, structured perfectly with clear explanation of all essential ideas. The talk should be delivered in a confident manner with respect to the audience.
Select Assignment 2 Type	Attendance
Input Example(s) of Assignment 2 (preferable)	To participate in the discussions during seminars, to understand what is going on