Skoltech
Skolkovo Institute of Science and Technology

Course Syllabus

Course Title (in English)	Symmetric functions
Course Title (in Russian)	Симметрические функции
Lead Instructor(s)	Olshanski, Grigori

Contact Person	Grigori Olshanski
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Course Description

The theory of symmetric functions has numerous applications in various domains of mathematics and mathematical physics. At the beginning of the course, standard material will be presented, and then we will move on to more advanced topics.

Tentative program:

The algebra Sym of symmetric functions. Generators of Sym. The scalar product, involution map, and Hopf algebra structure. Schur functions, skew Schur functions. Combinatorial formula. Cauchy identity and dual Cauchy identity. Jacobi-Trudi formula and its dual version.
Frobenius coordinates. Giambelli formula. Symmetric group characters. Murnaghan-Nakayama rule. Polynomial functions on Young diagrams. The Gessel-Viennot method. Supersymmetric functions. Interpolation symmetric polynomials. Multidimensional symmetric orthogonal polynomials. Generalized Schur polynomials and Macdonald's "9th variation". Beyond Schur polynomials (if time permits): Hall-Littlewood polynomials and other generalizations.

Course Prerequisites / Recommendations	Good working knowledge of the university course of algebra. Initial facts from representation theory of finite groups.

Аннотация

Теория симметрических функций имеет многочисленные приложения в различных областях математики и математической физики. В начале курса будет представлен стандартный материал, а затем мы перейдем к более продвинутым вопросам.

Предварительная программа:

Алгебра Sym симметрических функций. Генераторы алгебры Sym. Скалярное произведение, отображение инволюции и структура алгебры Хопфа. Функции Шура, косые функции Шура, комбинаторная формула. Тождество Коши, дуальное тождество Коши. Формула Якоби-Труди и ее дуальная версия. Координаты Фробениуса. Формула Джамбелли. Характеры симметрических групп. Правило Мурнагана-Накаямы. Полиномиальные функции на диаграммах Юнга. Метод Гесселя-Вьенно. Суперсимметрические функции. Интерполяционные симметрические полиномы. Многомерные симметрические ортогональные полиномы. Обобщенные полиномы Шура и «9-я вариация» Макдональда. За пределами полиномов Шура (если позволит время): полиномы Холла-Литтлвуда и другие обобщения.

Цель курса --- научить студентов технике работы с симметрическими функциями и дать подготовку к чтению специальной литературы.

Course Academic Level	Master-level course suitable for PhD students
Number of ECTS credits	6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Schur functions: basic facts	The algebra Sym of symmetric functions. Generators of Sym. The scalar product, involution map, and Hopf structure. Schur functions, skew Schur functions. Combinatorial formula. Cauchy identity and dual Cauchy identity. Jacobi-Trudi formula and its dual version.	10	7	
Young diagrams and symmetric group characters	Frobenius coordinates. Giambelli formula. Symmetric group characters. Murnaghan-Nakayama rule. Polynomial functions on Young diagrams.	9	7	
Supplements	The Gessel-Viennot method. Supersymmetric functions. Interpolation symmetric polynomials. Multidimensional symmetric orthogonal polynomials. Generalized Schur polynomials and Macdonald's "9th variation". Beyond Schur polynomials (if time permits): Hall-Littlewood polynomials and other generalizations.	9	7	

Assignment Type	Assignment Summary
Homework Assignments	exercises covering the whole material

Type of Assessment	Graded
Grade Structure	

Activity Type	Activity weight, %
Homework Assignments	100

A:	80
B:	70
C:	60
D:	50
E:	40
F:	0
Attendance Requirements	Optional

Maximum Number of Students

	Maximum Number of Students
Overall:	20
Per Group (for seminars and labs):	20

Course Stream	Science, Technology and Engineering (STE)
Course Term (in context of Academic Year)	Term 1 Term 2
Course Delivery Frequency	n/a

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Mathematical and Theoretical Physics	Mathematics and Mechanics

Course Tags	Math

Required Textbooks	ISBN-13 (or ISBN-10)
n/a	

Recommended Textbooks	ISBN-13 (or ISBN-10)
I. G. Macdonald. Symmetric functions and Hall polynomials (Russian translation of the first edition available).	0198534892
R. P. Stanley, Enumerative combinatorics, Vol. 2 (Russian translation available)	0521789877

Papers	DOI or URL
A. Okounkov, G. Olshanski. Shifted Schur functions. St. Petersburg Mathematical Journal, 1998, 9:2, 239–300 (Russian version: Algebra i Analiz, 9:2 (1997), 73–146)	http://mi.mathnet.ru/eng/aa762

Equipment
n/a

Software
n/a

Labs for Education	n/a

Knowledge
Fundamental results about symmetric functions

Skill
Working knowledge of fundamental results and constructions in the theory of symmetric functions

Experience
Experience of working with various symmetric functions

Select Assignment 1 Type	Homework Assignments
Input Example(s) of Assignment 1 (preferable)	1. Let h_n denote the n-th complete homogeneous function; compute h_n(1,q,q^2,q^3,...), where |q|<1. Do the same for the n-th elementary symmetric function e_n. 2. Compute the image under coproduct for given examples of symmetric functions. 3. Let x_1, x_2, ... be an infinite collection of formal variables; expand the product prod_{i 4. Write the Frobenius formula for the dimension of a Young diagram as an identity for a rational function and prove it using the method of residues.
Assessment Criteria for Assignment 1	The problems will vary in difficulty. The total score is calculated according to the formula min(100, 200*S/N), where S denotes the total number of points obtained and N denotes the maximal possible number of points