Course Syllabus

Course Title (in English)	Phase Transitions: Introduction to Statistical Physics and Percolation
Course Title (in Russian)	Фазовые переходы: введение в статистическую физику и перколяцию
Lead Instructor(s)	Shlosman, Semen

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1. Annotation

Course Description

This is a course on rigorous results in statistical mechanics, random fields, and percolation theory. We start with percolation, which is the simplest system, exhibiting singular behavior, and undergoing phase transitions. We then go to more realistic models of interacting particles, like the Ising model and XY-model, and study phase transitions, occurring there.
The topics will include:
Percolation models, infinite clusters.
Crossing probabilities for rectangles
Critical percolation
The Russo-Seymour-Welsh theory
Cardy's formula in Carleson form and the Smirnov theorem.
Gibbs distribution
Dobrushin-Lanford-Ruelle equation
Ising model
Spontaneous symmetry breaking at low temperatures
O(N)-symmetric models
The Mermin–Wagner Theorem
The Berezinskii–Kosterlitz–Thouless transition
Reflection Positivity and the chessboard estimates
Infrared bounds

Course Prerequisites / Recommendations	It is desirable that the students are familiar with the main notions of probability theory, measure theory, and functional analysis. Of course, the calculus knowledge is assumed.

Аннотация

В этом курсе я расскажу о строгих математических результатах статистической физики и теории просачивания, которые рассматриваются с точки зрения случайных полей.
Теория просачивания – простейшая из систем, в которой происходят фазовые переходы. Соответствующее случайное поле – это просто система независимых случайных величин. Мы увидим, что в критической точке корреляции убывают степенным образом, а все неё – экспоненциально.
Потом мы перейдём к системам с взаимодействием, таким как модель Изинга и XY-модель, и изучим фазовые переходы, происходящие в них. Мы увидим, какую роль в фазовых переходах играет группа симметрии взаимодействия.

2. Structure and Content

Course Academic Level	Master-level course suitable for PhD students
Number of ECTS credits	6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
The model of percolation.	Bond and site percolation on various lattices. Clusters, finite and infinite. Dual models, circuits. No percolation for small p. Percolation for p close to 1.	1	3	0
Crossing probabilities	The behavior of the probabilities of having a left-right crossing in the rectangle box. Subcritical, supercritical and critical case.	1	3	0
Critical percolation	The value of the critical probability. Sharpness of the criticality. Power-law decay of connectivity at the critical point.	1	3	0
The Russo-Seymour-Welsh (RSW) theory	Crossing probabilities for rectangles with arbitrary aspect ratio. Application to the connectivity estimates and to conformal invariance.	1	3	0
Cardy’s formula in Carleson form and the Smirnov theorem.	A reminder about the holomorphic functions, Cauchy–Riemann conditions and Riemann theorem. The concept of conformal invariance.	1	3	0
Gibbs distribution, the Ising model, DLR equation.	Interactions, Hamiltonians, boundary conditions, partition functions, finite volume Gibbs states.	1	3	0
Thermodynamic limit.	Free energy, Van Hove theorem, (in)dependence of free energy on the shape of the box and boundary conditions. Infinite volume Gibbs state.	1	3	0
One-dimensional models.	Markov chains and 1D Gibbs fields. Phase transitions in 1D systems.	1	3	0
Spontaneous symmetry breaking at low temperatures,	Peierls estimate. Contours. Positivity of magnetisation at low temperatures.	1	3	0
Dobrushin Uniqueness Theorem. Constructive uniqueness.	Uniqueness of the Gibbs state at high temperatures and in non-zero magnetic field.	1	3	0

3. Assignments

Assignment Type	Assignment Summary
Homework	I will ask the participants to solve simple problems. A volunteer will be asked to explain the solution.
Presentation	A topic will be suggested to be prepared and explained during a part of the lecture (15 min).
Report	A written report on a given theme can be counted towards the final exam.

4. Grading

Type of Assessment	Pass/Fail

Grade Structure

Activity Type	Activity weight, %
Final Exam	40
Class participation	20
Presentation	10
Homework Assignments	20
Report	10

Grading Scale

Pass:	70
Attendance Requirements	Mandatory with Exceptions

5. Basic Information

Maximum Number of Students

	Maximum Number of Students
Overall:	20
Per Group (for seminars and labs):	20

Course Stream	Science, Technology and Engineering (STE)
Course Term (in context of Academic Year)	Term 3

Course Delivery Frequency	Every two years

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Mathematical and Theoretical Physics	Mathematics and Mechanics

Course Tags	Math

6. Textbooks and Internet Resources

Required Textbooks	ISBN-13 (or ISBN-10)
Theory of Phase Transitions: Rigorous Results by Ya. G. Sinai	9780080264691
Statistical Mechanics of Lattice Systems: A Concrete Mathematical Introduction by Sacha Friedli, Yvan Velenik Cambridge University Press, 2017	9781107184824

Recommended Textbooks	ISBN-13 (or ISBN-10)
Statistical Physics L D Landau E.M. Lifshitz	9780080570464
Percolation, Second Edition by Geoffrey Grimmett Grundlehren der mathematischen Wissenschaften, vol 321, Springer, 1999	9783642084423

Papers	DOI or URL
Hugo Duminil-Copin Introduction to Bernoulli percolation	https://www.ihes.fr/~duminil/publi/2017percolation.pdf

Web-resources (links)	Description
http://www.unige.ch/math/folks/velenik/smbook/index.html	Statistical Mechanics of Lattice Systems: a Concrete Mathematical Introduction

7. Facilities

Equipment
laptop

Software
Mathematica

8. Learning Outcomes

Knowledge
Phase transitions. Percolation theory. Critical phenomena. Correlation decay, critical exponents. Theory of random fields, in particular, Markov fields and Gibbs fields. Mathematical theory of phase transitions. Random surfaces.

Skill
Ability to read and understand the literature on probability theory, percolation, rigorous statistical physics, e.g. Journal of Statistical Physics and (some) papers in Communications in Mathematical Physics.
Ability to formulate and sometime also solve problems in the theory of phase transitions and related areas.

Experience
Experience of working in the area of {mathematical physics}cap{probability theory}=ProbaΦ

9. Assessment Criteria

Input or Upload Example(s) of Assigment 1:

Select Assignment 1 Type	Homework Assignments
Input Example(s) of Assignment 1 (preferable)	Prove the existence of the phase transition for the Ising model on uniform (infinite) Cayley tree.
Or Upload Example(s) of Assignment 1	Prove the existence of phase transition for the Ising model on uniform (infinite) Cayley tree.
Assessment Criteria for Assignment 1	A - for the correct proof. For wrong or incomplete proof - depending on the sketch.

Input or Upload Example(s) of Assigment 2:

Select Assignment 2 Type	Presentation

Input Example(s) of Assignment 2 (preferable)	Explain the proof of the power-law decay of the percolation probability at criticality, filling in the details in the Duminil-Copin paper.
Assessment Criteria for Assignment 2	A - for clear clean unaided exposition, le A - depending on the amount of help needed.

Input or Upload Example(s) of Assigment 3:

Select Assignment 3 Type	Report
Input Example(s) of Assignment 3 (preferable)	Present a proof of the statement that the probability of crossing a rectangle stays away from 0 and 1.
Assessment Criteria for Assignment 3	A - for a complete proof. le A - depending on the completeness of the argument

Input or Upload Example(s) of Assigment 4:

Select Assignment 4 Type	Final Exam
Input Example(s) of Assignment 4 (preferable)	Prove the Margulis-Russo identity, using coupling.
Assessment Criteria for Assignment 4	A - for a complete proof. le A - depending on the completeness of the argument

Input or Upload Example(s) of Assigment 5:

10. Additional Notes