Skoltech
Skolkovo Institute of Science and Technology

Course Syllabus

Course Title (in English)	Introduction to quantum groups
Course Title (in Russian)	Введение в квантовые группы
Lead Instructor(s)	Bershtein, Mikhail
Is this syllabus complete, or do you plan to edit it again before sending it to the Education Office?	The syllabus is a final draft waiting for approval (once approved the syllabus will be published on the public web-site and other systems)
Contact Person	Mikhail Bershtein
Contact Person's E-mail	m.bershtein@skoltech.ru

1. Annotation

Course Description

Quantum groups were introduced in the mid-80's and very quickly became one of the most important topics in mathematics and mathematical physics. They are still actively studied, and their knowledge is necessary for work in many areas.

The purpose of the course is an introduction to quantum groups. The content will be based on classic works of the 80's and early 90's, we will not get to the latest results. Initial knowledge about quantum groups is not assumed, but acquaintance with Lie algebras and Groups, Poisson brackets, and the first notions of category theory is assumed.

Course Prerequisites / Recommendations

Students should be familiar with

1. Simple Lie algebras, universal enveloping algebras,Chevalley generators, Serre relations.
2. Lie groups, Poisson structures.
3. Basic notions of category theory.

Аннотация

Квантовые группы появились в середине 80-х годов и очень быстро стали одной из важнейших тем в математике и математической физике. Они до сих пор активно изучаются, а также их знание является необходимым для работы во многих областях науки.

Целью курса является введение в квантовые группы. Содержание будет основано на классических работах 80-х и начала 90-х годов, до последний результатов мы не дойдем. Начальных знаний про квантовые группы не предполагается, но предполагается знакомство с алгебрами и группами Ли, скобками Пуассона, первыми понятиями теории категорий.

2. Structure and Content

Course Academic Level	Master-level course suitable for PhD students
Number of ECTS credits	6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Poisson algebras and quantization	Poisson algebras. Quantization of Poisson algebras. Examples. *Quantization of Poisson manifolds.	1	2	2
Classical r-matrix and Poisson-Lie groups.	Classical r-matrix. Classical Yang-Baxter equation. Examples. Manin triples. Poisson-Lie groups, Lie bialgebras.	1	2	2
Example of sl(2)	Definition of Hopf algebras. Quantum universal envoloping sl_2, definitng relations, coproduct, R matrix. Functions on quantum group SL(2).	1	2	2
Tensor categories	Monoidal categories. Tensor categories. Braided categories. Coboundary, quasitriangular, triangular Hopf algebras. Quantum Yang-Baxter equation.	2	4	4
Drinfeld-Jimbo quantum group	Quantum borel subalgebra. Selfduality. Quantum Drinfeld double. Finite dimensional example.	1	2	2
W algebras and quantum groups	W algebras for type A. Screening operators. Quantum group relations.	1	2	2
RTT realization.	RTT definition of functions on quantum group. RTT definition of quantum universal enveloping algebra, equivalence of the definitions. RTT and Drinfeld double.	1	2	2
Lusztig group.	Lusztig braid group action. Cartan-Chevalley basis.	1	2	2
Universal R-matrix.	Universal R-matrix. Khoroshkin-Tolstoy formula. Center of the quauntum group.	2	4	4

3. Assignments

Assignment Type	Assignment Summary
Problem Set	List of problems and exercises, after each lecture

4. Grading

Type of Assessment	Graded
Grade Structure	

Activity Type	Activity weight, %
Class participation	50
Problem Set	50

Grading Scale

A:	86
B:	76
C:	66
D:	56
E:	46
F:	0
Attendance Requirements	Mandatory with Exceptions

5. Basic Information

Course Stream	Science, Technology and Engineering (STE)
Course Term (in context of Academic Year)	Term 1 Term 2
Course Delivery Frequency	Other

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Mathematical and Theoretical Physics	Mathematics and Mechanics

Course Tags	Math Physics

6. Textbooks and Internet Resources

Required Textbooks	ISBN-13 (or ISBN-10)
Etingof Pavel, Schiffman Olivier , "Lectures on Quantum Groups"	9781571462077
Pavel Etingof. Shlomo Gelaki. Dmitri Nikshych. Victor Ostrik. "Tensor Categories"	9781470420246

Recommended Textbooks	ISBN-13 (or ISBN-10)
Klimyk, Anatoli, Schmüdgen, Konrad Quantum groups and their representations	9783642646010

Papers	DOI or URL
V. G. Drinfeld, “Almost cocommutative Hopf algebras"	
Jin Tai Ding and Igor B. Frenkel "Isomorphism of two realizations of quantum affine algebra Uq(germggerml(n))"	10.1007/BF02098484
L. D. Faddeev , N. Yu. Reshetikhin, and L . A. Takhtajan "Quantization of Lie Groups and Lie Algebras"	
S.M. Khoroshkin, V.N. Tolstoy "Universal R-matrix for quantized (super)algebras"	10.1007/BF02102819

7. Facilities

8. Learning Outcomes

Knowledge
Basic notions on the theory of the quantum groups: Poisson-Lie groups, R-matrix, Yang-Baxter equation, Drinfeld double

Skill
Computations in noncommutative algebras similar to universal enveloping to Lie algebras. Reasoning and computation with Hopf algebras.

Experience
Translation of problems from a generic language of category theory to a concrete language of representation theory. Comparison of the quantum and classical pictures in the algebra.

9. Assessment Criteria

Input or Upload Example(s) of Assigment 1:

Select Assignment 1 Type	Problem Set
Input Example(s) of Assignment 1 (preferable)	Show equivalence between RTT and Drinfeld-Jimbo realization of quantum group gl(n).
Assessment Criteria for Assignment 1	Each task will be evaluated in the same number of points, so the total points for all tasks without asterisks is 50 The solutions of the problems can be discussed with any member of the instructional staf

Input or Upload Example(s) of Assigment 2:

Input or Upload Example(s) of Assigment 3:

Input or Upload Example(s) of Assigment 4:

Input or Upload Example(s) of Assigment 5:

10. Additional Notes