Skoltech
Skolkovo Institute of Science and Technology

Course Title (in English)	Quantum Integrable Systems
Course Title (in Russian)	Квантовые интегрируемые системы
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1. Annotation

Course Description

The course is devoted to quantum integrable systems. The history of quantum integrable systems starts from 1931 when
H.Bethe managed to construct exact eigenfunctions of the Hamiltonian
of the Heisenberg spin chain with the help of a special substitution
which became famous since that time (ansatz Bethe). In one or another
form this method turns out to be applicable to many spin and field-theoretical
integrable models. From the mathematical point of view, Bethe's method is
connected to representation theory of quantum algebras (q-deformations of
universal enveloping algebras and Yangians).

Here is the list of topics which will be discussed in the course.

- Coordinate Bethe ansatz on the example of the Heisenberg model and
one-dimensional Boe gas with point-like interaction between particles.

- Bethe ansatz in exactly solvable models of statistical mechanics
on the lattice.

- Calculation of physical quantities in integrable models in thermodynamic
limit, thermodynamic Bethe ansatz.

- Bethe equations and the Yang-Yang function, caclulation of norms of Bethe
vectors.

- Quantum inverse scattering method and algebraic Bethe ansatz, quantum R-matrices,
transfer matrices, Yang-Baxter equation.

- Functional Bethe ansatz and the method of Baxter's Q-operators, functional
relations for transfer matrices, transfer matrices as classical tau-functions.

The knowledge of quantum mechanics and statistical physics for understanding of
the course is highly desirable but not absolutely necessary. Out of the physical
context ansatz Bethe in its finite-dimensional version is simply a method
for diagonalization of big matrices of a special form. In this sense it does not
require anything except the basic notions of linear algebra.

Course Prerequisites / Recommendations	Students should have basic knowledge of linear algebra, quantum mechanics and statistical physics.
Аннотация	

Этот курс посвящен квантовым интегрируемым системам. История квантовых интегрируемых систем началась в 1931 году, когда Г.Бете построил точные волновые функции гамильтониана спиновой цепочки Гейзенберга с помощью специальной подстановки, котоая стала с тех пор знаменитой (анзац Бете). В том или ином виде этот метод оказывается применимым ко многим спиновым и теоретико-полевым интегрируемым моделям. С математической точки зрения метод Бете связан с теорией представлений квантовых алгебр (q-деформаций универсальных обертывающих алгебр и янгианов.

Вот список тем, которые будут затронуты в курсе.

- Координатный анзац Бете на примере модели Гейзенберга и одномерного Бозе-Газа с точечным взаимодействием между частицами.

- Анзац Бете в точно решаемых моделях статистической механики на решетке.

- Вычисление физических величин в интегрируемых моделях в термодинамическом пределе, термодинамический анзац Бете.

- Уравнения Бете и функция Янга-Янга, нормы векторов Бете.

- Квантовый метод обратной задачи и алгебраический анзац Бете, квантовые R-матрицы, трансфер-матрицы, уравнение Янга-Бакстера.

- Функциональный анзац Бете и метод Q-оператора Бакстера, функциональные соотношения для трансфер-матриц, трансфер-матрицы как клсссические тау-функции.

Знание квантовой механики и статистической физики весьма желательно для понимания курса, но не абсолютно необходимо. Вне физического контекста анзац Бете в его конечномерном варианте - это просто метод диагонализации больших матриц специального вида. В этом смысле он не требует ничего кроме владения основными понятиями линейной алгебры.

2. Structure and Content

Course Academic Level	Master-level
Number of ECTS credits	6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Coordinate Bethe ansatz	Coordinate Bethe ansatz in the spin chain (Heisenberg magnet) and in one-dimensional Bose gas. Bethe wave function. Bethe equations.	6	0	0
Bethe ansatz in exactly solvable models of statistical mechanics on the lattice	Bethe ansatz in exactly solvable models of statistical mechanics on the lattice. Six-vertex model.	6	0	0
Calculation of physical quantities in integrable models in thermodynamic limit	Calculation of thermodynamic quantities in integrable models in thermodynamic limit. Thermodynamic Bethe ansatz. Yang's integral equation.	6		
Quantum inverse scattering method	Quantum inverse scattering method and algebraic Bethe ansatz, quantum R-matrices, transfer matrices, Yang-Baxter equation	4	0	0
Functional Bethe ansatz	Functional Bethe ansatz and the method of Baxter's Q-operators, functional relations for transfer matrices, transfer matrices as classical tau-functions.	4	0	0
Quantum-classical correspondence	Functional relations for quantum transfer matrices as the mKP hierarchy of classical integrable equations	2		

3. Assignments

Assignment Type	Assignment Summary
Problem Set	A set of problems on the course.
Homework	A set of problems for solving at home.

4. Grading

Type of Assessment	Graded

Grade Structure		

Activity Type	Activity weight, %
Homework Assignments	20
Class participation	20
Problem Set	60

Grading Scale

A:	86

B:	76

C:	66

D:	56

E:	46

F:	0

Attendance Requirements	Mandatory with Exceptions

5. Basic Information

Maximum Number of Students

	Maximum Number of Students
Overall:	7
Per Group (for seminars and labs):	7

Course Stream	Science, Technology and Engineering (STE)
Course Term (in context of Academic Year)	Term 3 Term 4
Course Delivery Frequency	Every two years

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Mathematical and Theoretical Physics	

Course Tags	Math Physics

6. Textbooks and Internet Resources

Required Textbooks	ISBN-13 (or ISBN-10)
N.M.Bogoliubov, A.G.Izergin, V.E.Korepin, Correlation functions of integrable systems and quantum inverse scattering method, Moscow, Nauka, 1992	5-02-014626-7

Recommended Textbooks	ISBN-13 (or ISBN-10)
R.Baxter, Exactly solved models in statistical mechanics, Academic Press, 1982	9780486462714

Web-resources (links)	Description
http://people.sissa.it/ ffranchi/BAnotes.pdf	Notes on Bethe ansatz

7. Facilities

8. Learning Outcomes

Knowledge
Students will be familiar with general principles of the theory of quantum integrable systems

Skill
Students will be able to calculate physical quantities in quantum integrable systems

Experience
Students will get experience of dealing with quantum integrable systems and exactly solvable models of statistical mechanics

9. Assessment Criteria

Input or Upload Example(s) of Assigment 1:

Select Assignment 1 Type	Problem Set
Assessment Criteria for Assignment 1	Correct written solutions of most of the given problems. Ability to explain the solution in conversation.

Input or Upload Example(s) of Assigment 2:

Input or Upload Example(s) of Assigment 3:

Input or Upload Example(s) of Assigment 4:

Input or Upload Example(s) of Assigment 5:

10. Additional Notes