Skoltech
Skolkovo Institute of Science and Technology

Course Syllabus

Course Title (in English)	Elliptic Operators in Topology of Manifolds
Course Title (in Russian)	Эллиптические операторы в топологии многообразий
Lead Instructor(s)	Gaifullin, Alexander

Contact Person	Alexander Gaifullin
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1. Annotation

Course Description

The course will be devoted to applications of elliptic differential operators in topology of manifolds. We start with basics of Hodge theory including a detailed proof of the existence of the harmonic representative in a de Rham cohomology class. This proof is based on the usage of Sobolev spaces, and we will pay attention to this technique. Further, we proceed with the notion of the index of an elliptic differential operator towards the Atiyah-Singer theorem and its applications. Examples of several most important operators will be studied in details.

Course Prerequisites / Recommendations	Students should be familiar with the standard course of differential geometry, including differential forms and de Rham cohomology. Besides, students should be familiar with theory of vector bundles and connections in them. It is desirable that students know basic facts of functional analysis and partial differential equations.
Аннотация	

Курс будет посвящен приложениям эллиптических дифференциальных операторов в топологии многообразий. Мы начнем с основ теории Ходжа, включая подробное доказательство существования гармонического представителя в классе когомологий де Рама. Это доказательство опирается на использование пространств Соболева и мы уделим довольно много внимания этой технике. Далее курс продолжится обсуждением понятия индекса эллиптического оператора в направлении теоремы Атьи-Зингера и ее приложений. Будут подробно разобраны примеры нескольких наиболее важных операторов.

2. Structure and Content

Course Academic Level	Master-level course suitable for PhD students
Number of ECTS credits	6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Hodge theory	Hodge theorem: Existence of a harmonic representative in a de Rham cohomology class. Sobolev spaces.	5	18	
Atiyah-Singer theorem	Elliptic differential operators in vector bundles. Examples. Index of an elliptic operators. Elliptic complexes. Atiyah-Singer theorem. Its applications.	6	20	

3. Assignments

4. Grading

Type of Assessment	Graded

Grade Structure	

Activity Type	Activity weight, %
Final Exam	100

Grading Scale

A:	86
B:	76
C:	66
D:	56
E:	46
F:	0
Attendance Requirements	Mandatory with Exceptions

5. Basic Information

Course Term (in context of Academic Year)	Term 1-2
Course Delivery Frequency	Every year

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Mathematical and Theoretical Physics	Mathematics and Mechanics Physics

Course Tags	Math

6. Textbooks and Internet Resources

Required Textbooks	ISBN-13 (or ISBN-10)
Wells, RO, Differential analysis on complex manifolds, 1973	
Palais, RS, Seminar on the Atiyah-Singer index theorem, 1965	

Recommended Textbooks	ISBN-13 (or ISBN-10)
Griffits, P, Harris, J, Principles of algebraic geometry	
Л. Шварц, Комплексные аналитические многообразия. Эллиптические уравнения с частными производными, М.: Мир, 1982	

7. Facilities

8. Learning Outcomes

Knowledge
Hodge theory. Atiyah-Singer theorem.

Skill
Usage of elliptic complexes in topology of manifods.

Experience
Experience of applying analytic methods (Sobolev spaces, differential equations, etc.) in topology.

9. Assessment Criteria

Input or Upload Example(s) of Assigment 1:

Input or Upload Example(s) of Assigment 2:

Input or Upload Example(s) of Assigment 3:

Input or Upload Example(s) of Assigment 4:

Input or Upload Example(s) of Assigment 5:

10. Additional Notes