Skoltech
Skolkovo Institute of Science and Technology

Course Syllabus

Course Title (in English)	Representations of Classical Groups and Related Topics
Course Title (in Russian)	Представления классических групп и связанные с ними вопросы
Lead Instructor(s)	Grigori Olshanski

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1. Annotation

Course Description

The course is focused on fundamental results of the representation theory of classical matrix groups, which find numerous applications in various domains of mathematics. Particular attention will be paid to links with algebraic combinatorics.

Tentative program:

- Characters of classical groups (general linear, orthogonal, and symplectic)
- Second Weyl character formula
- Classical invariant theory and applications
- Representations in traceless tensors
- Brauer duality
- Highest weight representations and Littlewood formulas
- Center of universal enveloping algebra
- Perelomov-Popov theorem
- Capelli identity
- Multidimensional interpolation polynomials
- Binomial formula for characters
- Okounkov's quantum immanants
- Applications to asymptotic representation theory

Курс нацелен на фундаментальные результаты теории представлений классических матричных групп, имеющие многочисленные применения в различных областях математики. Особое внимание будет уделено связям с алгебраической комбинаторикой.

Примерная программа:

- Характеры классических групп (полных линейных, ортогональных и симплектических)
- Вторая формула Вейля для характеров
- Классическая теория инвариантов и приложения
- Представления в бесследовых тензорах
- Двойственность Брауэра
- Представления со старшим весом и формулы Литтлвуда
- Центр универсальной обертывающей алгебры
- Теорема Переломова-Попова
- Тождество Капелли
- Многомерные интерполяционные полиномы
- Биномиальная формула для характеров
- Квантовые иммананты Окунькова
- Применения к асимптотической теории представлений

Course Prerequisites / Recommendations	Good knowledge of basic algebra (groups, rings, modules). Good knowledge of linear algebra. Basics of Lie groups/Lie algebra theory. Some acquaintance with basics of general representation theory of finite/compact groups would be helpful. Basics of functional analysis (Hilbert space, bounded operators on Hilbert space).

Аннотация

Курс нацелен на фундаментальные результаты теории представлений классических матричных групп, имеющие многочисленные применения в различных областях математики. Особое внимание будет уделено связям с алгебраической комбинаторикой.

Задача курса -- научить студентов пользоваться аппаратом комбинаторной теории представлений.

Основные разделы программы:
- Конечномерные представления и характеры классических групп
- Результаты, связанные с универсальными обертывающими алгебрами классических алгебр Ли
- Многомерные интерполяционные полиномы и их применения

2. Structure and Content

Course Academic Level	Master-level course suitable for PhD students
Number of ECTS credits	3

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Finite-dimensional representations and characters	- Characters of classical groups (general linear, orthogonal, and symplectic). - Second Weyl character formulas - Classical invariant theory and applications - Representations in traceless tensors - Brauer duality - Highest weight representations and Littlewood formulas	27	-	-
Universal unveloping algebra and its center	- Highest weight representations - Center of universal enveloping algebra - Perelomov-Popov theorem - Capelli identity	27		
Multidimensional interpolation polynomials and applications	- Multidimensional interpolation polynomials - Binomial formula for characters - Okounkov's quantum immanants - Applications to asymptotic representation theory	27		

3. Assignments

Assignment Type	Assignment Summary
Problem Set	exercises covering the whole material

4. Grading

Type of Assessment	Graded

Grade Structure	

Activity Type	Activity weight, %
Homework Assignments	-2

Grading Scale

A:	80
B:	70
C:	60
D:	50
E:	40
F:	0
Attendance Requirements	Optional

5. Basic Information

Maximum Number of Students

	Maximum Number of Students
Overall:	30
Per Group (for seminars and labs):	

Course Stream	Science, Technology and Engineering (STE)
Course Term (in context of Academic Year)	Term 1 Term 2
Course Delivery Frequency	n/a

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Mathematical and Theoretical Physics	Mathematics and Mechanics

Course Tags	Math

6. Textbooks and Internet Resources

Required Textbooks	ISBN-13 (or ISBN-10)
n/a	

Recommended Textbooks	ISBN-13 (or ISBN-10)
Hermann Weyl. The classical groups. Their invariants and representations (Russian translation available)	9780691057569
D.P.Zhelobenko. Compact Lie groups and their representations (Translation of Russian edition)	9780821815908

Papers	DOI or URL
A.Okounkov and G.Olshanski, Shifted Schur functions	https://arxiv.org/abs/q-alg/9605042
A.Okounkov and G.Olshanski, Shifted Schur functions II. Binomial formula for characters of classical groups and applications	https://arxiv.org/abs/q-alg/9612025

7. Facilities

8. Learning Outcomes

Knowledge
Fundamental results about finite-dimensional representations of classical groups, their characters, and related topics.

Skill
Working knowledge of fundamental results and constructions in combinatorial representation theory.

Experience
Experience of working with characters of classical groups.

9. Assessment Criteria

Input or Upload Example(s) of Assigment 1:

Select Assignment 1 Type	Homework Assignments
Input Example(s) of Assignment 1 (preferable)	1. Compute the dimension of an irreducible representation with a given highest weight. 2. Decompose a given representation into irreducibles. 3. Deduce the Jacobi-Trudi formula from the Cauchy identity
Assessment Criteria for Assignment 1	The problems will vary in difficulty. Each solved problem may give 1, 2 or 3 points. The total score is calculated according to the formula min(100, 200*S/N), where S denotes the total number of points obtained and N denotes the maximal possible number of points.

Input or Upload Example(s) of Assigment 2:

Input or Upload Example(s) of Assigment 3:

Input or Upload Example(s) of Assigment 4:

Input or Upload Example(s) of Assigment 5:

10. Additional Notes