\[ \psi(qa) = S(a,q) \psi(a) \]

\[ \{ \frac{2\pi i}{a} \} = \text{weights of } A \text{ in } N_{X/X^A} \mathcal{U} \]
More precisely,

Vertex function for $T^d$

$$\sum_{\lambda} \frac{1}{(\nu_{\lambda})^d (\nu_{\lambda}^* a)^d} \quad \text{rational if } \nu = \nu_{\lambda}^*.$$

$$1 \pmod{(\nu - 1)}$$

$$\Rightarrow \quad \text{on the Kähler side there will be Kähler roots } \subset \text{characters } \mathbb{Z}_{H^2(X, \mathbb{Z})}$$

Recall,

Among all difference equations in equivariant variables, we want to find $q^{K^Z}$

$$S_{\sigma} = \langle \sigma \rangle$$

we count with $\mathbb{Z}^{\text{deg } a}$ as usual

formal power series in $\mathbb{Z}$.

$$\Psi(q_{a_1, a_2}) = (\mathbb{Z} \otimes 1) R_{a_2}(a_1/a_2) \Psi$$

no $q$ here
\( <w, \sigma > = w \) \rightsquigarrow \text{get singularities at } \sqrt{q} \wedge \psi \text{ has only singularities at } q^n, n \in \mathbb{Z}

\text{Typically, not the case}

\text{Has to be the case that } |<\sigma, \text{root}>| \leq 1.

\text{Means } \sigma \text{ has to be minuscule}

\( \sigma \) acts with weights \( \pm 1, 0 \)

\( X \subset \text{vector space} \)

\( X_0 \subset \text{functions on } X \)

functions have generated functions on \( X_0 \)

**Def.** \( \sigma \) is minuscule if \( C[X] \) is generated by functions of weight \( 0, \pm 1 \) w.r.t. to \( \sigma \)

**Exercise.**

\( \mathbf{6}(b) = \begin{pmatrix} b & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \in GL(W) \subset Aut(Nakajima, w) \) is minuscule.
Means $S_\circ$ has a chance to be $qk\mathbb{Z}$

Back to $S_\circ = \sigma$.

$\Psi(qa_1, a_2) = (z \otimes 1) R_{12} (a_1/a_2) \Psi$.

$z$ appears only outside as a monomial.

No $q$ here.

Only "constant" quasi-maps contribute.

All other contributions will vanish by rigidity.

Properness is important.

The result is a Laurent poly.

Properness + bound the weights.

The Newton polygon of that Laurent poly contains no lattice points.

Silly example: $X = \mathbb{C}^2$, $\sigma(b) = \begin{pmatrix} b^k & 0 \\ 0 & b^{-k} \end{pmatrix} \in \text{Aut}(X, \omega)$.

$X = \sigma^\infty X = \mathcal{O}(k) \oplus \mathcal{O}(-k)$, $k > 0$. 

(properness)
can restrict the values of the section at 0 and \(\infty\).

First approximation:

\[
\begin{array}{c}
\text{class on } X \\
\rightarrow \\
\text{class on } X
\end{array}
\]

\[
H^0(O(k-\mathbb{D}[-\mathbb{D}]))
\]

\(k > 1\).

\(k = 1\) proper!

Not great when we will start counting weights

\[
\mathbb{C}^2
\]

\[
\delta_t = (1-b^{-1})(1-b)
\]

Newton polygon ( ) = \([-1, 1]\)

We can do better

The section can't go in this direction anymore

\[
H^0(O(k-\mathbb{D}[-\mathbb{D}]))
\]

\(k > 1\).
Thm

\[ K(X^0) \xrightarrow{\text{Stab}_-} K(X^0) \]

\[ K(X) \xrightarrow{\text{Stab}_-} K(X) \]

1. is proper
2. vanishes for non-constant maps

actual work [pcmi] Section 10.2

\[ \text{Stab}_{\text{transp}} = (\text{Stab}_+)^{-1}. \]

For general \( X \), proper \( X \xrightarrow{\text{proper}} X^0 \xrightarrow{\text{proper}} \text{vector space with weights } 0, \pm 1. \)

6-twisted \( \text{QM}(X) \) proper \( \xrightarrow{\text{proper}} \) sections of \( O(1)^{t_0} \otimes O(-1)^{t_1} \otimes \cdots \otimes O. \)

\( f \)-twisted \( \text{QM}(\text{vector space}). \)

For constant maps, set \( \pm 2 \text{ dim}. \)

Properness here is like in the silly example.

\( \) matrix is \( \text{Stab}_{\rightarrow} \to \text{Stab}_{\rightarrow} \) we get a diagram.
$\text{K}(X^0) \xrightarrow{qKZ} \text{K}(X^0) \xrightarrow{\text{Stab}_+} \text{K}(X)$

$\text{Stab}_+ = (\text{Stab}_-)^T \cdot \text{Stab}_+$

from this, one compute difference equations in \( \text{K"ahler variables} \Rightarrow \text{next time} \)

What about non-minuscule shifts \( \sigma \)?

in \( qKZ \)

There is a natural conjecture

\[
S_0 = \prod \text{R-matrices for } X^a
\]

all root hyperplanes crossed \( \sigma \)

also a Nekyjima quiver variety

Good progress by Y. Kononov and A. Smirnov