Basic pieces of enumerative theory of $\mathcal{C} = \mathbb{P}^1$:

- Evaluation to $\mathcal{E}$
- Maps nonsingular at this point, evaluation to $\mathcal{X}$
- Relative point, accords open, proper evaluation to $\mathcal{X}$

By degeneration: $\mathcal{C} = \infty$ and $\mathcal{C} = \infty$.

The matrix that connects $\mathcal{C}$ and $\mathcal{C}$ is $p(n) \times p(n)$ matrix.
By localization: \( \text{ev} \rightarrow \text{ev}_0 \)

**In fact:** \( \text{ev}_0 \rightarrow \text{ev}_0 \)

So, a bunch of interrelated tensors in \( K(\mathcal{X}) \odot^2 \) or \( K(\mathcal{X}) \odot K(\mathcal{X}) \), ...

\[ \begin{align*}
0 \rightarrow \text{view as} \quad & e_{\lambda^*} (e_{\nu^*} (\cdot) \cdot \hat{\Theta}_{\text{vir}} z^\text{deg}) \in K(\mathcal{X}) \odot K(\mathcal{X}) \odot [\mathbb{C}] \\
& \text{K(X)} \\
& \text{convergent}
\end{align*} \]

Claim: This map, with suitable \( q \)-Gamma function factors is the elliptic stable envelope in the context of \( X \subset \mathcal{X} \).

**A class on the open in this instance**

\[ \text{Attr} \subset \text{component of } X^A \times (X \setminus \text{lower attracting manifolds}) \]

Now: GIT stable locus

\[ X \subset \mathcal{X} \]
$\mathcal{X} \setminus X = \text{has a Bogomolov-Hesselink-Kempf-Ness-Rousseau-... stratification}$

quotient by $G$, e.g. $G = \text{NGL}(V)$

reductive connected (will explain)

may be reduced to tori by a trick

restriction

$\text{Stab} : \quad \bigcirc \left( T^{1/2} \text{prequotient} \right) \otimes \bigcirc (T^{1/2} \text{prequotient}) \otimes \bigcirc (T^{1/2} \text{prequotient}) \otimes \cdots \rightarrow \bigcirc (T^{1/2} \text{prequotient}) \otimes \bigcirc (T^{1/2} \text{prequotient})$

sections of line bundle on $\text{Ell}_G \times \text{G}_{\text{aut}} (\mu_t)$

$\mathcal{X} = \left[ \frac{\text{prequotient}}{G} \right]$ $\text{Ell}_{\text{equiv}} (X)$

for $E = \mathbb{C}^*/q \mathbb{Z}$

our $q$

means: interpolate an elliptic function from finitely many values

$\text{is a fundamental solution of a flat } q \text{-difference connection}$

in both Kähler variables $z \in \text{Pic}(X) \otimes \mathbb{C}^*$

and equivariant variables in $\text{Aut}(X)$

This is for $U_1(\mathbb{Z})$

is not related to $q$

$\xrightarrow{a \mapsto \sigma(q) a}$

a generalization of $q \mathbb{Z}$

to a cocharacter $\sigma : \mathbb{C}^* \rightarrow \text{Aut}(X)$

Very classical question: can one solve this equation by an $\int$?

in usual hypergeometric world "Euler" $\int_0^1 x^{a-1} (1-x)^{b-1} (1-x^q)^{-c} \ dx$

in $q$-difference situation "Mellin-Barnes" $\int \text{poly}(x) \frac{\Gamma(n)}{\Gamma(n-q)} x^q \ dx$
in a difference situation

\[ (1-x)^{\infty} \sim (1-x)(1-qx) \ldots (1-q^{m-1}x) = \frac{(x)^{x}}{(q^{m}x)^{x}} \quad \text{character of } \mathcal{C} \left[ \text{generators of weight } x, qx, q^{x}, \ldots \right] \]

\[ \Gamma_{q}(x) = \frac{1}{\prod_{i=0}^{\infty} (1-q^{i}x)} \quad \text{coordinate on } \mathcal{C} \]

\[ \text{Maps}(\mathcal{C} \rightarrow \mathcal{C}) \quad \text{cycle} (\beta) \]

\[ \text{Elliptic function} \]

\[ \text{in our situation, this takes the form:} \]

\[ \text{from Maps}(\mathcal{C} \rightarrow \text{Prequantum}) \]

\[ \otimes_{o} = \otimes_{o} \text{solution of } \Gamma_{q} \]

This is my full solution.

If I can find \( f_{\beta} \) such that

\[ \text{then } \alpha(\otimes_{o}) = f_{\beta}(\otimes_{o}) \]

solved by stable envelopes in \( K \)-theory, so

\[ \star, \otimes, \text{deg} \]
Then
\[ a \circ 0 = \circ 0 \]
\[ \sigma \circ \rho = f_{a} \circ \rho = \chi \left( QM, e_{\nu}^{*} (f_{a}) e_{\nu}^{*} (\beta) \hat{G}_{\nu} \right) \frac{dx}{\nu_{i} x_{i}}. \]

The origin of \( \Gamma_{q} \) factors in the formula.

\[ = \chi \left( \mathcal{X}, f_{d} \otimes \text{elliptic stable} (\beta) \otimes e_{\nu_{i} x_{i}} \text{(maps) } \mathcal{F} \rightarrow \mathcal{X} \right) \]

\[ = \int f_{d}(x) g_{q}(x) \Gamma_{q} \text{-functions } dx. \]

Suppose
\[ \int f_{d}(x) g_{q}(x, z) \text{ weight } \prod_{i=1}^{n} \frac{dx_{i}}{\nu_{i} x_{i}} \]

The eigenvector solves an equation of the form
\[ \Psi(qz) = M(z) \Psi(z) \]

What will happen if \( q \to 1 \)
\[ \Psi(z) \sim e^{\frac{i}{n} \sum_{i} \lambda_{i}(z)} \Psi_{i}(z) \]

This limit tells me both eigenvalues and eigenvectors of \( M(z) \mid_{q=1} \)

\[ f_{d}(x) = \text{insertion at 0} \]
\[ \int f_d(x, \ldots) \, g_{\beta}(x, \ldots) \, \text{weight} \prod_{i=1}^{2n} x_i^2 \]

\( |x_i| = 1 \)

For \( \text{Hilb}(\mathbb{C}^2, n) \) they

\[ \varphi(x) \varphi(t x_i / x_j) \varphi(q x_i / t x_j) \]

Singularity is here.

Killed by \( g_{\beta} \)

because \( g_{\beta} \) is supported on \( \mu^{-1}(0) \subset \mathbb{C} \)

\[ g_{\beta} |_{\mathbb{C} \setminus \mu^{-1}(0)} = 0 \]

\( \text{Ell}(\mathbb{C} \setminus \mu^{-1}(0)) \hookrightarrow \text{Ell}(\mathbb{C}) \)

"wheel conditions"

Elliptic curve of \( K \)-theory

\[ \mathbb{C}^* \rightarrow \mathbb{C}^*/q\mathbb{Z} = E \]

\( \forall X, G \) \[ \text{Spec } K_G(X) \rightarrow \text{Ell}_G(X) \]

"reduction mod \( q \)"

Solution of some abelian q-difference eq.

Section of line bundle

\[ \psi(x) \text{ as a function of } x \in \mathbb{C}^* \text{ satisfies} \]

\[ \psi(x) = -q^2 x^{-1} \psi(x) \]
$V(x)$ as a function of $x \in \mathcal{A}$ satisfies

something like

$V(qx) = -q^{-2} x^{-1} \theta(x)$