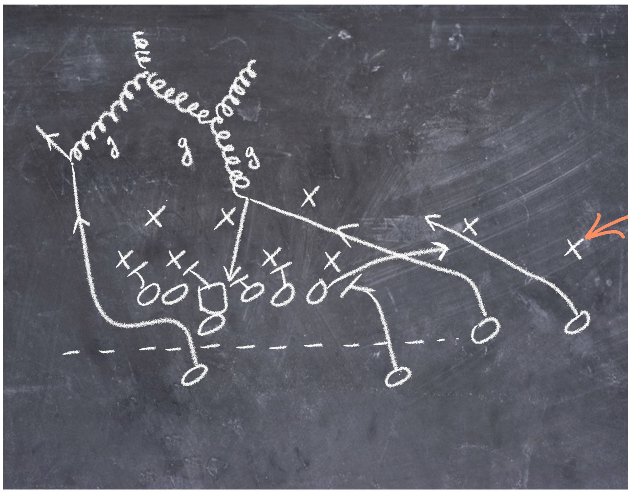
Lecture 23, 10/20

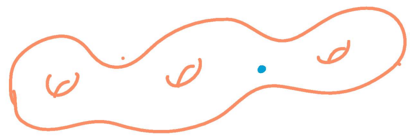
Monday, October 19, 2020 5:40 PM



Enumerative geometry & geometric representation theory

Start time Moscow 17:30
New York 10:30

base pt.



a component of the source curve C

base point

a point, evaluates to the stack $\mathfrak{X}$

$\cup$ stable

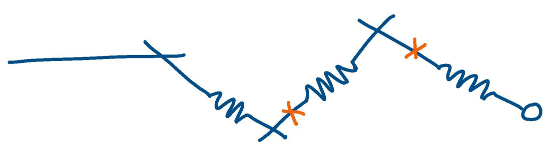
open

a nonsingular point, evaluates to X

a $\mathbb{P}^1$ component / $\mathbb{C}^*$ rescaling

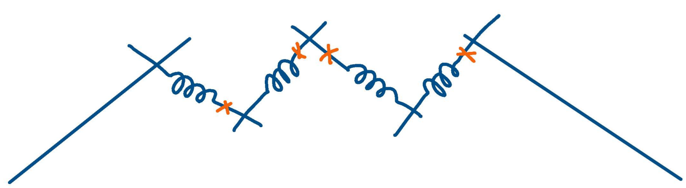
bubble

really =

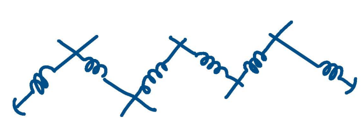


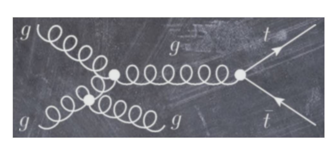
a relative point, proper evaluation to X

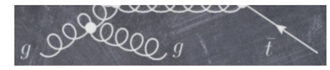
really =



Glue = (spring) really =

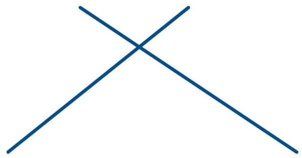




Glue =  =  

accordions, not like that

<u>Thm</u> (read the proof in penn notes 6.5)

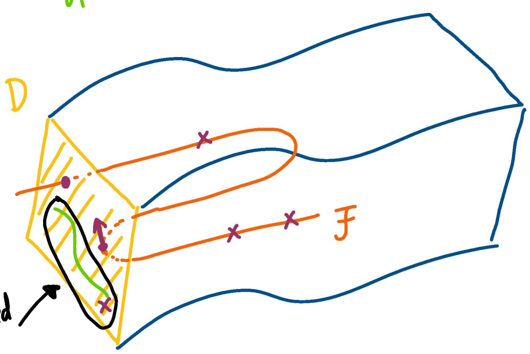
 = $\longrightarrow$ Glue^{-1} $\longleftarrow$

shorthand

a divisor in the moduli of source curves

as K-theory classes on that divisor

D



$\mathcal{F}$

3-fold Y e.g. $\mathbb{C}^2 \hookrightarrow Y$
$\downarrow$
C

disallowed

no components in the divisor

had no 0-dimensional subschemes

$$
\begin{tikzcd}
\mathcal{O}_Y \arrow[r, "s"] \arrow[d] & \mathcal{F} \arrow[d] \\
\mathcal{O}_D \arrow[r, "s \bmod x"] & \mathcal{F} \otimes \mathcal{O}_D
\end{tikzcd}
$$

evaluation map

generates if there are no base points in the divisor

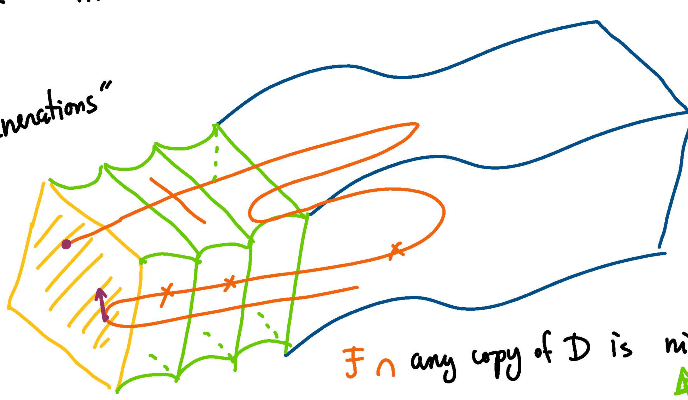
 means a divisor D

normal direction to D

in 

$\longleftarrow$ means

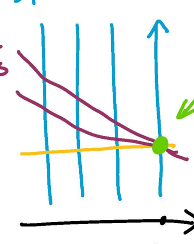
Jun Li "expanded degenerations"



$\mathcal{F} \cap$ any copy of D is nice

space

base pts



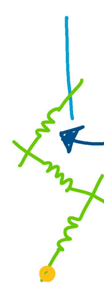
blow $D \times \{0\}$

Divisor $D \times$ time.

$t=0$ time

space time

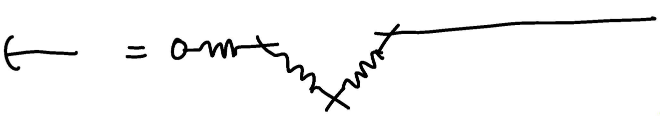
$\downarrow$ $\downarrow$



$\mathbb{P}(N_{Y/D} \oplus \mathcal{O}_D)$

bubble

$\mathbb{P}^1$ bundle over D with 2 sections

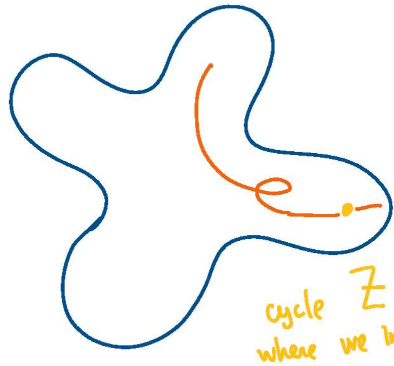
$\longleftarrow$ = 

quasimap should be nonsingular at the node.

Y

the line. with

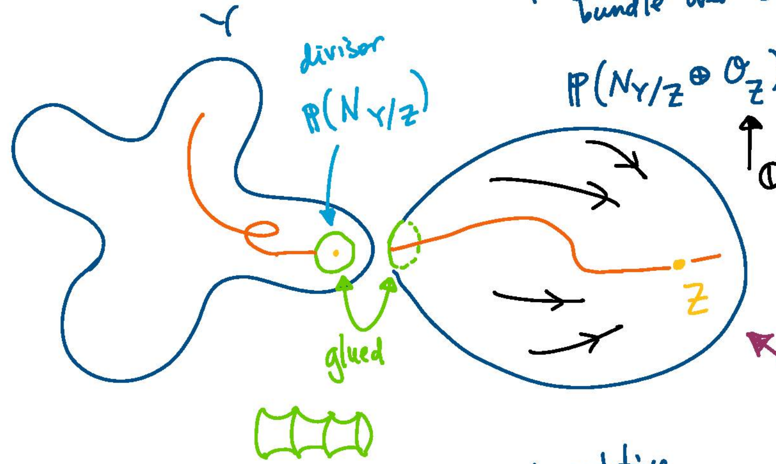
Y



cycle Z
where we impose
some geometric cond.

still can blow up $Z \times \{t=0\}$ in $Y \times$ time
get a new 3-fold

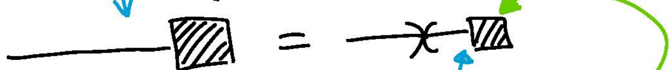
projective space
bundle over Z



counts relative
to a divisor in Y
+
whatever counts I
wanted originally in
a model geometry

anything

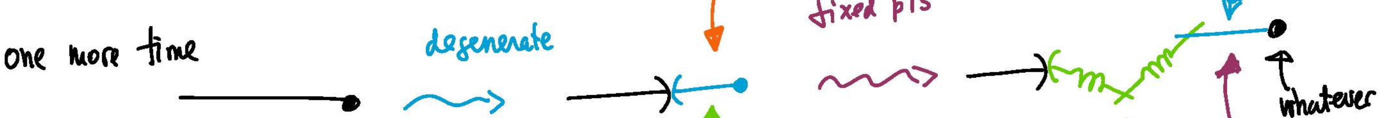
whatever
geometric conditions



still parametrized

here constant
and nonsing.

fixed pts



bubbles

analog of taking tubular
neighborhood.

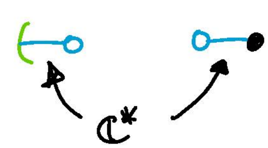
has $\mathbb{C}^*$
action

localise.

here $\mathbb{C}^*$
does not act.

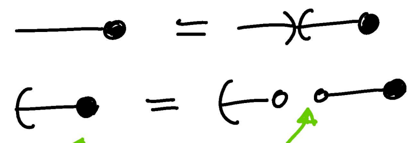
break up
along here

$\mathbb{C}^*$ acts.



node, there are bubbles
in which this
does not act

∃ relations between "boundary conditions"

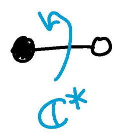
∃ translations between "boundary conditions" 

anything →

$(o\!-\!o)^{-1}$. simple.

<u>Take away</u>:

if X is one of the varieties we consider then, it is important to know tensors of the form

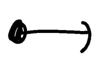
 $\in K_{eq}(\mathcal{X}) \otimes K_{eq}(X) \otimes$ series in z ← counts the degree

$\mathbb{C}^*$

 $\underset{\text{Thm}}{=}$ Glue

$\mathbb{P}^1$

 $\in K_{eq}(X)^{\otimes 2} \otimes \ldots$

 $\in \ldots$

<u>Goal</u> describe all of these in terms of quantum groups acting

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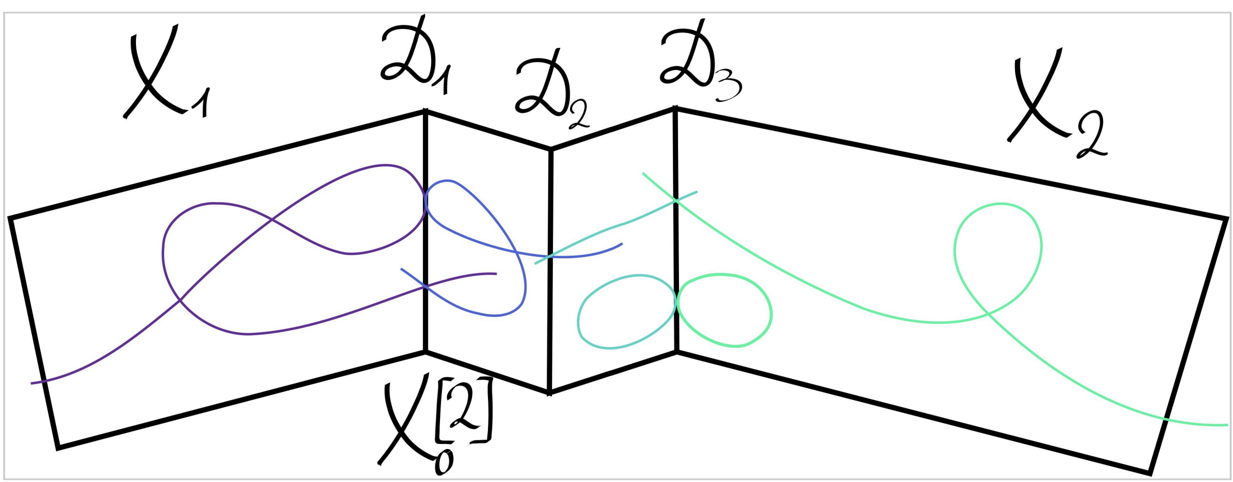
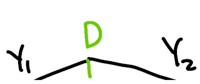
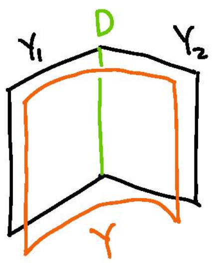


More about relative conditions and degeneration



$Y \rightsquigarrow Y_1 \cup_D Y_2$

$N_{Y_1/D} \otimes N_{Y_2/D} = \mathcal{O}_D$



$Y \rightsquigarrow Y_1 \cup_D Y_2$ $\qquad N_{Y_1/D} \otimes N_{Y_2/D} = \mathcal{O}_D$

<u>Thm</u> (Levine–Pandharipande) $[Y] = [Y_1] + [Y_2] - [\text{bubble } \mathbb{B}]$, $\quad \mathbb{B} = \mathbb{P}(\mathcal{O}_D \oplus N_{Y_1/D})$

generates <u>algebraic cobordism</u>

over $pt = \operatorname{Spec} \mathbb{C}$ $=$ Lazard ring

$\otimes \mathbb{Q} = \mathbb{Q}[\mathbb{P}^1, \mathbb{P}^2, \mathbb{P}^3, \dots]$

does not immediately reduce all counts to toric, but still....

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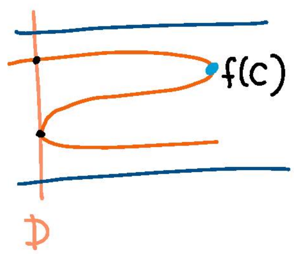
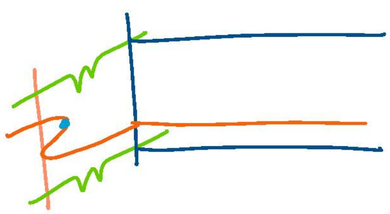
in GW theory



$f^*D = \sum_i \mu_i [p_i]$

fixed



evaluation map $\in$ Orbifold cohomology of

$\exp(D) = \bigsqcup_n D^n / S(n)$.

BKR+Haiman

$\bigcup_n \mathrm{Hilb}(D, n)$

General conjecture (known for ADE fibrations over curves), in cohomology

relative counts in GW $=\!=$ relative counts in PT/DT/...

Mumford

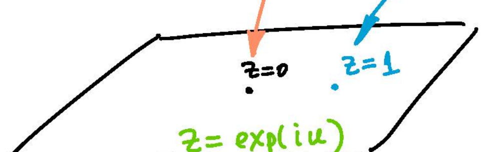
Gromov–Witten of target curves with relative insertions is very nice, e.g. has Virasoro

$\longleftrightarrow$ very, very close to thing like DRC

$0 \quad \infty$

zeros $\quad$ pole

Hilb$(\mathbb{C}^2, n)$ $\qquad$ orbifold $\mathbb{C}^{2n}/S(n)$.



z counts the degree $\in \operatorname{Pic}(X) \otimes \mathbb{C}^* = \mathbb{C}^*$ for Hilb.

counts the genus in GW