$X \simeq \text{moduli space of vacua} \supset \text{Grit}(\Phi)/U$ compact

$M = \text{"matter"} = \text{a symplectic representation of } G \supset U$ complex

$X = M // G = \mu_1^+(0)//G = \mu_1^+(0,\theta)/U$

$\mu_1 = \mu_G \oplus \mu_R \rightarrow \mathfrak{g}^* \otimes (C \oplus R)$

$\phi = \langle \mu_1^+, \xi_1^+ \rangle, \quad \xi_1^+ \in \mathfrak{g} \otimes (C \oplus R)$

$a$ very rich world of examples

$X$ is smooth, is rare

e.g. no stabilizers in the $\mu_1^-(0) \cap$ semistable

For $G = \text{TTGL}(V_i)$

$M = V_i \oplus \omega_i \oplus \text{dual}$

$\text{Hom}(V_i, V_j) \oplus \text{dual}$

$X = \text{Nakajima quiver variety}$

$G$ commutes with $\text{TTGL}(W_i) \times \text{TTGL}(E_{ij})$

fixed points here are $W_i$

TT quiver varieties of same type

$\text{Hom}(V_i, V_j) \rightarrow \text{dual}$

$\text{GL}(V_i)$ does not act here

$\text{M has to be small}$

$\text{Hom}(W_i, V_i)$

$\text{GL}(V_i)$ does not act here

fixed points of these elements are quiver varieties with

$\tilde{Q} \rightarrow Q$

$Q =$ (no arrows)

\[ \text{e.g. } Q = \bullet \]
\[ \Theta W_i \quad \text{TI quiver vanishing in same type} \quad \text{e.g.} \quad Q = \cdot \quad (\text{no arrows}) \]

\[ W = W' \oplus a W'' \]

\[ X(w) = \bigoplus_v X(v, w) \quad X(w) = T_\infty(w) = \bigcup_{\nu} T^{[\nu]}(\nu, w) \]

\[ X(w) = X(w') \times X(w^*) \quad \iff \]

\[ R(a) \quad R \text{-matrix satisfying YB eq. etc.} \quad a_2 = a_3 \quad X(w') \times X(w'' - w'''') \]

\[ R_{23} \]

\[ \rightarrow \text{a quantum group} \quad \text{Goal: this quantum group is an elliptic deformation } U(\hat{g}) \]

\[ \hat{g} = \text{Lie algebra} \quad \text{loops into } \hat{g} \]

\[ \hat{g} \approx \text{Lie algebra } \quad \text{loops in } \mathfrak{gl}(1) \]

\[ \hat{g}_{\text{KM}} = \mathfrak{sl}(3) \quad \mathfrak{gl}(1) \]

\[ \hat{g} = \mathfrak{o}(3) \]

\[ \hat{g}_0 \quad \hat{g}_2 \quad \hat{g}_0 \quad \hat{g}_2 \quad \hat{g}_0 \quad \hat{g}_2 \quad \hat{g}_0 \quad \hat{g}_2 \quad \hat{g}_0 \quad \hat{g}_2 \]

\[ \mathfrak{gl}(1) \quad \mathfrak{sl}(3) \quad \text{rotation central extension} \]

\[ [\hat{g}_n, \hat{g}_m] = n \delta_{n+m} \quad \text{for enumerative applications} \]

\[ \hat{g}_{\text{KM}} = \langle \hat{g}_{n+1}, \frac{d}{dt}, c \rangle \]

\[ \hat{g} \approx \hat{g}_{\text{KM}} \quad \text{the difference is much more dramatic for other quivers} \quad \hat{g} \gg \hat{g}_{\text{KM}} \]

\[ \text{For } Q \quad X(1) = \bigsqcup_{n \geq 0} \text{Hilb}(C^2, n) \quad \text{and } X(r) = \text{moduli space of certain sheaves on } C^2 \]

\[ \text{of rank } r \]
ideal $I \subset \mathbb{C}[x_1, x_2]$ of codimension $n$

Goal: understand the quantum group as a deformation of $U(g)$

Elliptic quantum group $\rightarrow U_q(g) \rightarrow \mathcal{Y}(g) =$ a graded deformation of $U(g)$

Curve $E$ of $\text{Ell}_1(X)$

here, stable envelope was a section of some line bundle $\mathcal{L}$

$\text{Stab} \xrightarrow{\sim} \text{section of a certain line bundle on } \text{Ell}_A(\text{pt}) = E$

se rank $A$

abelian variety

$\mathcal{L} \rightarrow \mathcal{X} = \begin{array}{c} \text{have different size, according to deg } \mathcal{L} \\ \text{toric variety + ample line bundle } \leftrightarrow \text{polytope} \end{array}$

$\mathfrak{h} A = 1 \Rightarrow H^*_c(X, \mathbb{Z})$

"periodic" toric variety

all these pieces $\xrightarrow{K_c^*(X)} K_c^*(X, \mathbb{Z}) / \mathbb{Z}^2$
Shtak

Surface Simple:

\[ \text{deg} = \frac{1}{2} \text{codim} \]

\[ \text{deg} < \frac{1}{2} \text{codim} \]

Evidence and uniqueness for shtak envelope in \( H^*(X) \) is rather easy.

Newton:

\[ (\text{degree} \alpha (x) < \text{degree} \alpha (T)) \]

\[ = \frac{1}{2} \text{codim} F_i \]

\[ \text{Newton}(x \cdot T/2) \]

Has to fit inside Newton inside sections are monomials that fit.

Zero:

\[ E = Z/\Lambda = E(A) \]

\[ E = C^2/\Lambda = E(A) \]

In other words, blow up origin in the special fibre

\[ H^*(X) \]

\[ H^*(X) \]

\[ H^*(X) \]

\[ H^*(X) \]

\[ E \]

all those places have projective subspace, each associated to \( E \).
\[ R(u) = 1 + \frac{1}{u} r + \ldots \]

Classical r-matrix

\[ [r, (12)] = 0 \quad \text{r is symmetric} \]

\[ R_{12}(u_1-u_2) R_{13}(u_1-u_3) R_{23}(u_2-u_3) = R_{23} R_{13} R_{12} \]

Take \( \frac{1}{u_2} \) coefficient as \( u_1 \to \infty \):

\[ [r_{12} + r_{23}, R_{23}(u_2-u_3)] = 0 \]

In particular:

\[ [r_{12} + r_{13}, r_{23}] = 0 \quad \text{and symm.} \]

\[ [r_{13}, r_{23}] = - [r_{12}, r_{23}] \]

Take matrix elements in 1 and 2 \Rightarrow matrix elements form a Lie algebra of \( r \in \mathfrak{so}_2 \) is an invariant

The matrix \( r \) is an example of a Lagrangian Steinberg correspondence between \( X(w_1) \times X(w_2) \) and \( X(w_1') \times X(w_2') \) (where \( w_1 + w_2 = w_1' + w_2' \)).

A correspondence \( Z \subset X \times Y \)
\[ X = \text{Nakajima variety} = \ldots \text{//} G \]

\[ = \text{Proj} \oplus R_n \xrightarrow{\text{Spec} R_0 = X_0} \text{affine} \]

\[ \text{affine of invariants} \]

\[ X^A \text{ inclusion} \xrightarrow{} X \rightarrow X_0 \text{ affine.} \]

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In fact: Nakajima varieties are examples of symplectic resolutions, i.e. \( X \rightarrow X_0 \) is birational on its image.

For such \( X \), Steinberg variety \( X \times X \times X \) is isotropic.

In particular, \( \dim \text{Steinberg} \leq \dim X \)

\[ (1 \otimes \Delta) R = R_{12} R_{13} \]

\[ \Rightarrow (1 \otimes \Delta) r = r_{12} + r_{13} \text{, commutes not only with } r_{13}, \text{ but } R_{23}! \]

**General feature of Lagrangian Steinberg correspondences, they intertwine \( R \)-matrices.**

\[ \text{rational, that is in } H^*_T(...) \]