for $\text{GL}_n$ \( \otimes \) evaluation fun $\text{realized geometrically as}\)
\[
\bigwedge^k \mathbb{C}^n (a_i)
\]

\(q\)-quantum elliptic...

stable envelopes $\implies$ explicit formulas for offshell Bethe eigenfunctions

\[
\delta^2, \text{ quiver (no arrows)} \quad \otimes \mathbb{C}^2 (a_i) \quad \overset{\sim}{\longrightarrow} \quad T G(n) = \bigsqcup_{k=0}^{n} T^* \text{Grass}(k, n)
\]

\[A = (a_1, \ldots, a_n) \subset \text{GL}(n)\]

\(T G(n)^A = \text{coordinate subspaces} \subset \text{Grass} \subset T^* \text{Grass} \quad 2^n\)

\# \(T^* \text{Grass}(k, n)^A = \binom{n}{k}\)

\(\text{Gr}(k, n) = \left\{ V \subset \mathbb{C}^n \right\}\)

image of $\mathbb{C}^k \overset{\sim}{\longrightarrow} \mathbb{C}^n$

\(\text{Gr}(k, n) = \text{Hom} (\mathbb{C}^k \rightarrow \mathbb{C}^n) \overset{\text{rank } k}{\longrightarrow} / \text{GL}(k)\)

\(\begin{array}{c}
\delta
\end{array} \quad \begin{array}{c}
\text{of rank } k, \text{ modulo column operations}
\end{array} \quad \begin{array}{c}
\begin{array}{c}
\begin{array}{c}
1
\end{array}
\end{array}
\end{array}
\]

\(= \text{Gr}(k, n)^A\)

\(\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
1
\end{array}
\end{array}
\end{array}
\]

\(n^8 \quad \text{other...}\)
\[ A_{\text{tr}}(p) = \binom{1}{1,1,1} \cdot p = \binom{1}{n-k} \]

\[ \begin{array}{cccc}
4 & 5 & 4 & 5 \\
5 & 4 & 4 & 5 \\
4 & 5 & 4 & 5 \\
5 & 4 & 4 & 5 \\
\end{array} \]

complementary partition \( \lambda = (2,2,0) \)

\[ \lambda + \gamma = (4,3,0) \]

degree in cohomology

\[ |\lambda^c| = \text{dimension of } G_p \]

\[ n-k \]

\[ \beta \]

\[ \mathbb{C}^8 \] Schubert cell

arbitrary

\( \mathcal{S}_p = \text{Schubert}(p) \)

\[ \gamma \]

\[ \mathcal{H}^*(Gr) = \langle \text{Chern classes of the tautological bundle} \rangle = \text{Symm functions in } (x_1, x_2) / \text{ideal} \]

\[ \text{Answer: Schur function } S_\lambda(x_1, \ldots, x_k) = \text{Symm} \frac{\prod x_i + p_i}{\prod (x_i - x_j)} \]

\[ p = (k-\gamma', k-\gamma, 0) \]

\[ m_\lambda(x) = x^{\lambda} \quad a_i = 0 \]

or

\[ m_\lambda(x) = (x-a_1)(x-a_2)\ldots(x-a_\lambda) \]

Newton interpolation

\[ \sum_{\text{inter}} = [\mathcal{S}_\lambda] \text{ in } H^*_A(Gr) \]

Lascoux–Schützenberger

Proof: 1) has degree \( |\lambda| \)

2) vanishes at all fixed pts that are not in \( G_\lambda \)

\[ \sum_{\text{inter}} a^\mu = 0 \text{ unless } \lambda \subseteq \mu \]

\[ (a_{\lambda_1 + p_1}, a_{\lambda_2 + p_2}, \ldots) \]

characterization (up to multiple).

the unique poly of degree \( \ell \)
$m_l(x) = (x-a_1)(x-a_2) \cdots (x-a_l)$

the unique polynomial of degree $l$ that vanishes at $a_1, \ldots, a_l$

has a pole of a certain order at $x = \infty$

with this specialization one can do more

\[ \frac{1}{\hbar} \text{Symm} \prod_{i=1}^{n} m_{l_i}(x_i) \quad \text{rational function} \]

$T^* \text{Gr}(k, n)$ has degree

become Schubert closures $\Theta_\lambda$ when $\hbar \to \infty$

$T^* \text{Gr} \subseteq \mu^{-1}(0) \subseteq [\text{Hom} @ \text{Hom} / GL(n)]$

Spec $H^*(T^* \text{Gr}) \subseteq \text{Lie A} \times \text{Lie H}/W$

by Chern roots of the tautological bundle

$\mathfrak{gl}(k) = \text{J} \mathfrak{t}$

$\mathfrak{gl}(k) \to \mathfrak{gl}(k)/\mathfrak{t}$

$I$ injective as $T$ surjective
Elliptic version:

1. Take elliptic envelopes for $T^* \mathbb{P}^{n-1}$ as our functions $m_x$.

   
   $m_x(s) = \frac{\theta(a_1 s) \cdots \theta(a_{n-1} s)}{\theta(z^k \cdots)}$.

2. Symm $\frac{\prod \theta(x_i/x_j)}{\prod \theta(x_i/x_j \cdots)}$.

   
   $\text{Symm} \frac{\prod \theta(x_i/x_j)}{\prod \theta(x_i/x_j \cdots)}$.

   
   Proof $\implies$ D. Shenfeld (PhD thesis).

   
   $\mu_G^{-1}(o)/H \rightarrow \mu_H^{-1}(o)/H$.

   
   Push forward involves $1/\theta(x_i/x_j)$.

   
   $G = \text{GL}(k) \supset H$ = maximal torus.

Why only $i < j$? Look into actual geometry.

E.g., $\text{GL}(k)/H = \text{general conjugacy class} \cong T^* \text{Flag}$.
Very general, e.g., for \( \text{Hilb}(\mathcal{A}^2, k) \) formulas in [Smirnov].

For

\[
\mu_k(0)/\mathcal{C} \to (T^*\mathcal{C}(k, n))^k.
\]

Elliptic stable envelopes are flop-invariant and vanish outside of the full attracting set.

Elliptic cohomology is defined equivalent K-theory and equivariant cohomology.

Irreducible, genus 1, nodal, cuspidal.

\[ s \in E \text{ chern root of } \mathcal{O}(0) \]

E becomes \( \mathcal{C} = \text{Pic}_0(\text{cuspidal}) \).
\( s \in E \) \( \text{chern root of } O(0) \)

Stack envelope for \( \mathbb{P}^1 \mathbb{P}^{n-1} \) section of line bundle on \( E \) of degree \( n \)

\( a_1, a_2, \ldots, a_{k-1} \)

fixed pts

\( a_\text{top}, a_\text{top} \)

\( \text{Slop.} \)

\( \mathbb{P}^1 \text{Pic}^0(\text{cuspidal}) \)

\( s = \text{chern root of } O(1) \)

\( E \text{ becomes } \mathbb{C}^* \)

\( \text{chern root for } O(0) \)

\( k_{eq}(T^* \mathbb{P}^{n-1}) \)

\( \text{Fundamental} \)

Everything we do is \textbf{flat} in the elliptic curve.