Starting point: \( M_i(a) \leftarrow \text{vector space, a free module over } K[a] \text{ or } K[a^{\pm 1}] \) = Cohomology equivariant \((X)\) \( a \in G_m \subset \text{Aut}(X) \)

satisfies the YB equation.

Extend R-matrices to \( \widehat{\otimes} M_{ki}(a_i) \)

Step one:

Step two: define operators in

by taking matrix elements in

and values in \( v_i \) or any other linear functionals

Clearly an algebra: multiplication:

also comultiplication

needs a bit of a discussion if \( \dim M_i = \infty \)

e.g. for \( \mathfrak{u}_n(\mathfrak{o}(2)) \)
\[ M = \text{Cohomology}(X) \quad X = \bigsqcup_{k \geq 0} X_k \quad X_k = T^*\text{Grass}(k, \mathbb{C}^n) \]

for \( U_k(\mathfrak{g} \mathfrak{l}(n)) \)

\[ X_k = \text{Hilb}(\mathbb{C}^2, k) \]

= ideals in \( \mathbb{C}[x_1, x_2] \) of codimension \( k \)

one can take graded matrix elements to define \( U_k(\mathfrak{g}) \)

Comultiplication: \( U_k(\mathfrak{g}) \xrightarrow{\Delta} U_k(\mathfrak{g}) \otimes U_k(\mathfrak{g}) \)

Example \( a \in \mathcal{G}_{\text{add}}, \mathcal{G}_{\text{mult}}, \text{Elliptic curve} \)

\[ u \to \infty \]

\[ R(u) = 1 - \frac{r}{u} + O\left(\frac{1}{u^2}\right) \quad u \to \infty \]

\[ r = \sum e_i \otimes e^i \in S^2 \mathfrak{g} \]

\[ \mathfrak{g}_0 = \mathfrak{f} \oplus \bigoplus \mathfrak{g}_{\alpha} \]

\[ \text{Lie algebra with a nondeq invariant bilinear form} \]

\[ \Delta e_i = \ ? \]

\[ \Delta e_i = e_i \otimes 1 + 1 \otimes e_i \]

\[ [R, \Delta e_i] = 0 \quad R \text{ is } \mathfrak{g}_0 \text{-invariant} \]

\[ |\lambda \rangle \in \mathcal{M} \text{ be a lowest weight vector of weight } \lambda \]

\[ \mathfrak{g}_0 |\lambda \rangle = 0 \quad a < 0 \quad \mathfrak{h} \in \mathfrak{f} \quad \mathfrak{h} |\lambda \rangle = \lambda(\mathfrak{h}) |\lambda \rangle \]

\[ R_{\langle \lambda, \cdot \rangle} = 1 - \frac{h_1}{u} + \frac{h_2 \langle \cdot \rangle}{u^2} \]

\[ \lambda(\mathfrak{h}_i) h_i \]

deformation of \( t h_1 \)

in our deformation \( \mathcal{U}(\mathfrak{g} \left[ t \right]) \)
\[
\Delta h^{(1)}_\lambda = h^{(1)}_\lambda \otimes 1 + 1 \otimes h^{(1)}_\lambda - \sum_{\alpha > 0} (\lambda, \alpha) \, r^{-\alpha}
\]

\[
\langle \lambda | e_\beta e_\alpha | \lambda \rangle = \begin{cases} 
0 & \text{unless } \alpha + \beta = 0, \alpha > 0 \\
-(\lambda, \alpha) & \text{if } \alpha + \beta = 0, \alpha > 0
\end{cases}
\]

Two remarks:
1. Usually, the condition that \([R, \Delta \sigma_j] = 0\) and
\[
R \Delta h^{(1)}_\lambda = \Delta_{\text{opp}} h^{(1)}_\lambda \, R
\]
determine \(R\) uniquely up to a multiple.
2. Example \(\sigma_j = \sigma_j^{\text{gl}(1)} = \alpha_n\) \([\alpha_n, \alpha_m] = n \delta_{n+m} \, c \r \ldots + \sum_n \alpha_n \otimes \alpha_{-n} \)
bilinear expressions in \(\alpha_n\) \(\Delta^{(1)}_n = \text{Virasoro}\)
Sugawara-like formulas for Virasoro means \(R\) is the Liouville reflection operator

It is obvious that \(R\) do give the braiding of the modules we constructed
We can use in auxiliary space instead of the physical space!

\[ M_1 \quad M_2 \]

\[ \text{YB is both the commutation relation and cocommutation relation!} \]

We constructed a tensor category

\[ \text{Objects} = \bigotimes M_{\mathfrak{g}}(a_i) \]

\[ \text{Morphisms} = \text{maps that commute with } R \text{-matrices} \]

also a relation in \( U_q(\mathfrak{g}) \)

How to actually construct \( R \)-matrices?

\[ K_{eq}(X) \]

moduli of vacua in some \( \text{not} \) space

\[ X \quad x \quad x' \quad M \]

\[ \text{reductive } \]

\[ G \subset \text{GL}(V) \]

\[ \text{Weyl} \]

\[ \text{Hom}_G(V \otimes k \rightarrow V \otimes k) \]

\[ \text{critical pt} \]
a kindergarden illustration:

\[ X = pt \subset C^* \cong a \]

\[ K^*_q(X) = \mathbb{Z}[a^{\pm 1}] \]

pushforward \[ K^*_q(X) \rightarrow K^*_q(Y) \]

\[ O_0 = (1 - a^{-1}) O_C \]

Coker \( (K^*_q(X) \rightarrow K^*_q(Y)) = \) \[ \mathbb{Z}[a^{\pm 1}]/(1-a^{-1}) \]

Key example: \( U_h(\mathfrak{gl}(2)) \) on \( K^*_q(X_n) \), \( X_n = \bigsqcup_k T^* \text{Grass}(k,n) \)

\[ X_1 = pt \sqcup pt \]

\[ K^*_q(X_1) = \mathbb{Z}[h^{-1}, a_1^{\pm 1}] \otimes \mathbb{Z} \]
\[ K_{eq}(X_1) = U \{ \pi \} \]  

\text{“} C^2(\alpha) \text{”}

\[ X_2 = p^t \sqcup T^*P^1 \sqcup p^t \]  

by the weight of \( f \).

\[ T^*P^1 \]

\[ K_{eq}(2pt) \rightarrow K_{eq}(T^*P^1) \]

need to supply!

\[ K_{eq}(\text{Attr}_{a_2/a_2 \rightarrow 0}) \]

different in different cohomology theory

\[ a_2/a_2 \rightarrow 0 \]

\[ a_2 \rightarrow \infty \]

\[ \text{Attr.} \]

\[ X_3 = \bigsqcup \limits_k T^*\text{Grass}(k,3) \]

for every \( a \), consider \( X^a \)

\[ a_1 = a_2 \]

\[ a_1 \gg a_2 > a_3 \]

\[ X_4^3 \]

\[ X_5^3 \]

\[ a_2 = a_3 \]

Next time: intro to \( H_{eq}, K_{eq} \)