Skoltech
Skolkovo Institute of Science and Technology

Course Syllabus

Course Title (in English)	Quantum integrable systems
Course Title (in Russian)	Квантовые интегрируемые системы
Lead Instructor(s)	Zabrodin, Anton

Status of this Syllabus	The syllabus is a final draft waiting for form approval
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1. Annotation

Course Description

The course is devoted to quantum integrable systems. The history of quantum integrable systems starts from 1931 when
H.Bethe managed to construct exact eigenfunctions of the Hamiltonian of the Heisenberg spin chain with the help of a special substitution which became famous since that time (ansatz Bethe). In one or another form this method turns out to be applicable to many spin and field-theoretical integrable models. From the mathematical point of view, Bethe's method is connected to representation theory of quantum algebras (q-deformations of universal enveloping algebras and Yangians).

Here is the list of topics which will be discussed in the course.

- Coordinate Bethe ansatz on the example of the Heisenberg model and one-dimensional Boe gas with point-like interaction between particles.

- Bethe ansatz in exactly solvable models of statistical mechanics on the lattice.

- Calculation of physical quantities in integrable models in thermodynamic limit, thermodynamic Bethe ansatz.

- Bethe equations and the Yang-Yang function, caclulation of norms of Bethe vectors.

- Quantum inverse scattering method and algebraic Bethe ansatz, quantum R-matrices, transfer matrices, Yang-Baxter equation.

- Functional Bethe ansatz and the method of Baxter's Q-operators, functional relations for transfer matrices, transfer matrices as classical tau-functions.

The knowledge of quantum mechanics and statistical physics for understanding of the course is highly desirable but not absolutely necessary. Out of the physical context ansatz Bethe in its finite-dimensional version is simply a method for diagonalization of big matrices of a special form. In this sense it does not require anything except the basic notions of linear algebra.

Course Prerequisites	Students should have basic knowledge of linear algebra, quantum mechanics and statistical physics.

2. Structure and Content

Course Academic Level	Master-level
Number of ECTS credits	6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Coordinate Bethe ansatz	Coordinate Bethe ansatz in the spin chain (Heisenberg magnet) and in one-dimensional Bose gas. Bethe wave function. Bethe equations.	6	0	0
Bethe ansatz in exactly solvable models of statistical mechanics on the lattice	Bethe ansatz in exactly solvable models of statistical mechanics on the lattice. Six-vertex model.	6	0	0
Calculation of physical quantities in integrable models in thermodynamic limit	Calculation of thermodynamic quantities in integrable models in thermodynamic limit. Thermodynamic Bethe ansatz. Yang's integral equation.	6		
Quantum inverse scattering method	Quantum inverse scattering method and algebraic Bethe ansatz, quantum R-matrices, transfer matrices, Yang-Baxter equation	4	0	0
Functional Bethe ansatz	Functional Bethe ansatz and the method of Baxter's Q-operators, functional relations for transfer matrices, transfer matrices as classical tau-functions.	4	0	0
Quantum-classical correspondence	Functional relations for quantum transfer matrices as the mKP hierarchy of classical integrable equations	2		

3. Assignments

Assignment Type	Assignment Summary
Problem Set	A set of problems on the course.
Homework	A set of problems for solving at home.

4. Grading

Type of Assessment	Graded

Grade Structure

Activity Type	Activity weight, %
Homework Assignments	20
Class participation	20
Problem Set	60

Grading Scale

A:	86
B:	76
C:	66
D:	56
E:	46
F:	0
Attendance Requirements	Mandatory with Exceptions

5. Basic Information

Maximum Number of Students

	Maximum Number of Students
Overall:	7
Per Group (for seminars and labs):	7

Course Stream	Science, Technology and Engineering (STE)
Course Term (in context of Academic Year)	Term 3 Term 4
Course Delivery Frequency	Every two years

Students of Which Programs do You Recommend to Consider this Course as an Elective?

Masters Programs	PhD Programs
Mathematical and Theoretical Physics	

Course Tags	Math Physics

6. Textbooks and Internet Resources

Required Textbooks	ISBN-13 (or ISBN-10)
N.M.Bogoliubov, A.G.Izergin, V.E.Korepin, Correlation functions of integrable systems and quantum inverse scattering method, Moscow, Nauka, 1992	5-02-014626-7

Recommended Textbooks	ISBN-13 (or ISBN-10)
R.Baxter, Exactly solved models in statistical mechanics, Academic Press, 1982	9780486462714

Web-resources (links)	Description
http://people.sissa.it/ ffranchi/BAnotes.pdf	Notes on Bethe ansatz

7. Facilities

8. Learning Outcomes

Knowledge
Students will be familiar with general principles of the theory of quantum integrable systems

Skill
Students will be able to calculate physical quantities in quantum integrable systems

Experience
Students will get experience of dealing with quantum integrable systems and exactly solvable models of statistical mechanics

9. Assessment Criteria

Input or Upload Example(s) of Assigment 1:

Select Assignment 1 Type	Problem Set
Assessment Criteria for Assignment 1	Correct written solutions of most of the given problems. Ability to explain the solution in conversation.

Input or Upload Example(s) of Assigment 2:

Input or Upload Example(s) of Assigment 3:

Input or Upload Example(s) of Assigment 4:

Input or Upload Example(s) of Assigment 5:

10. Additional Notes