

Course Title (in English) Vertex operator algebras

Course Title (in Russian) Вертекс операторные алгебры

Lead Instructor(s) Feigin, Evgeny

Status of this Syllabus The syllabus is a final draft waiting for form approval

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1. Annotation

Course Description

Infinite-dimensional Lie algebras (such as Virasoro algebra or affine Kac-Moody algebras) turn out to be very important in various areas of modern mathematics and mathematical physics. In particular, they are very useful in the description of some field theories. In this context one arranges infinite number of the Lie algebra elements into a single object called field. This idea generalizes to the general theory of vertex operator algebras. VOAs capture the main properties of the infinite dimensional Lie algebras and have rich additional structure. Vertex operator algebras proved to be very useful in many situations; the classical example is the KP integrable hierarchy. They are also extensively used in modern algebraic geometry. Our goal is to give an introduction to the theory of vertex operator algebras from the modern mathematical point of view. We describe the main definitions, constructions and applications of the theory. The course is aimed at PhD students and master students.

Course Prerequisites Basic Lie theory, the theory of affine Kac-Moody algebras

2. Structure and Content

Course Academic Level Master-level course suitable for PhD students

Number of ECTS credits 6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Heisenberg algebras and Fock modules.	Representation theory of the Heisenberg algebra.	2	8	
Virasoro algebras, Verma modules.	Representation theory of the Virasoro algebra.	2	8	
Boson-fermion correspondence, Schur polynomials, KP hierarchy.	Introduction to the boson-fermion correspondence.	2	8	
Vertex operator algebras and Lie algebras.	Definitions and basic examples of vertex operator algebras.	2	8	
Representation theory of vertex operator algebras	Properties of VOAs and of representations of VOAs.	2	8	

3. Assignments

Assignment Type	Assignment Summary
Problem Set	Problems of the Heisenberg and Virasoro VOAs.
Problem Set	Problems on the operator product expansion of fields.
Problem Set	Problems on the structure of modules over the vertex operator algebras.

4. Grading

Type of Assessment	Graded	
Grade Structure	Activity Type	Activity weight, %
	Homework Assignments	15
	Exam/Quiz	40
	Midterm Exam	20
	Class Participation	25

Grading Scale

A:	80
B:	70
C:	60
D:	50
E:	40
F:	30

5. Basic Information

Attendance Requirements Optional

Maximum Number of Students	Maximum Number of Students	
	Overall:	15
	Per Group (for seminars and labs):	15

Course Stream Science, Technology and Engineering (STE)

Course Term (in context of Academic Year) Term 3
Term 4

Students of Which Programs do You Recommend to Consider this Course as an Elective?	Masters Programs	PhD Programs
	Mathematical and Theoretical Physics	Mathematics and Mechanics Physics

Course Tags Math
Physics

6. Textbooks and Internet Resources

Required Textbooks	ISBN-10 or ISBN-13
Kac V., Raina A., Rozhkovskaya N. Bombay lectures on Highest weight representations of infinite dimensional Lie algebras.	978-981-4522-19-9

Recommended Textbooks	ISBN-10 or ISBN-13
Frenkel E., Ben-Zwi D. Vertex algebras and algebraic curves	978-0-8218-3674-3

7. Facilities

8. Learning Outcomes

Knowledge
The structure theory and representation theory of vertex operator algebras.

Skill
Mathematical language of modern quantum field theory.

Experience
Working with various problems of representation theory and mathematical physics.

Do you want to specify outcomes in another framework? Knowledge-Skill-Experience is good enough

9. Assessment Criteria

Select Assignment 1 Type Homework

**Or Upload Example(s) of
Assignment 1**

<https://ucarecdn.com/37e8fb40-9ac3-43df-90a1-7ee53adeb7e6/>

**Assessment Criteria for
Assignment 1**

A: 80, B: 70, C: 60, D: 50, E: 40, F:30

Select Assignment 2 Type

Test/Quiz

**Or Upload Example(s) of
Assignment 2**

<https://ucarecdn.com/40169b72-1d30-41ba-97b8-4d9e4ac7833c/>

**Assessment Criteria for
Assignment 2**

A: 80, B: 70, C: 60, D: 50, E: 40, F:30

10. Additional Notes