

Problems for R. Bezrukavnikov's lecture 3
Skolkovo Summer School, July 2019

Problem 1. For $G = SL(2)$, $SL(3)$ show that there is a unique one dimensional Iwahori orbit on Gr . Find the dimension of the corresponding irreducible representation in the principal block in representations of Lusztig quantum group $Rep(U_q^{Lus})_0$ over \mathbb{C} and/or the category of modular representations $Rep(G)_0$.

Problem 2. Recall the action of the monoidal category $D_B(G/B)$ on $D_B(G/P)$ and on the partial Whittaker category $D_B(G/(U, \psi_P))$ for a character $\psi : U \rightarrow \mathbb{C}$. Identify the Grothendieck group of $D_B(G/B)$ with $\mathbb{Z}[W]$ by sending the class of extension by zero of the perverse constant sheaf from the orbit corresponding to w to the element w . Prove that the Grothendieck groups $K^0(D_B(G/P))$, $K^0(D_B(G/(U, \psi_P)))$ are induced from the trivial and the sign representations of $\mathbb{Z}[W_L]$ respectively where W_L is the Weyl group of the Levi.

- a) When $P = G$.
- b) When $G = SL(3)$.
- c) In general.