Problem 1. Let $G = GL(n)$. Prove classification of $G(O)$-orbits on the affine Grassmannian using Jordan normal form theorem from linear algebra. Deduce classification of Iwahori orbits.

Optional: describe the adjunction partial order on the set of orbits (and prove your answer).

Problem 2. Compute $IH^*(Q)$ where $Q \subset \mathbb{C}^n$ is the zero set of a nondegenerate homogeneous quadratic polynomial. Consider $n = 3$, 4.

Optional: try also to work out the case of an arbitrary $n > 2$.

Problem 3. Let $G = SL(2)$. Show that the closure of the two dimensional $G(O)$ orbit on the affine Grassmannian is locally near the singular point isomorphic to a quadratic cone. Compute its intersection cohomology with various coefficients. Verify directly the corresponding facts about representations of $PGL(2)$.

Problem 4. a) The affine Grassmannian of $PGL(n)$ has n components. Describe the closed $G(O)$ orbit on each component.

b) Show that the dimension of cohomology of such a closed orbit equals dimension of the fundamental representation $\Lambda^i(k^n)$ for some i.

c) Let h be a generator of the second cohomology of affine Grassmannian. Multiplication by h as an endomorphism of cohomology of a closed $G(O)$-orbit is conjugate to the action of a regular nilpotent $e \in sl(n)$ (i.e. the nilpotent with one Jordan block) on $\Lambda^i(k^n)$; verify this fact directly for $i = 1, 2$.

Optional: do the same for all i.

Problem 5. Prove (the corrected version of) the geometric statement from the sketch of a proof given in the lecture. Namely, show that for $G = SL(2)$ the map from the closure of a $G(\mathbb{C}[[t]])$ orbit on the affine flag variety $\mathcal{F}\ell$ to its image in the affine Grassmannian $\mathcal{G}r$ is semismall and the corresponding stratification of the image consists of two strata: the full image and a codimension two stratum.