Master test Skoltech

June 2018

1. Find the matrix for Casimir element $ef + fe + \frac{1}{2}h^2$ in three dimensional representation of $\mathfrak{sl}_2$.

2. Find exponent of the matrix $\begin{pmatrix} 2 & 1 \\ -3 & 2 \end{pmatrix}$.

3. Quantum harmonic oscillator with frequency ω was in the groud state. The frequency instantly changed to $\widetilde{\omega}$. What is the transition probability to the new ground state?

4. Find contour integral of $\dfrac{dz}{z^2 - 1}$ over the square with vertices $(2,0)$, $(0,2)$, $(-2,0)$, $(0,-2)$.

5. Solve differential equation on $\psi(x)$: $-\psi''(x) + (W(x)^2 + W'(x))\psi(x) = 0$

6. Find the number of sequences of integer numbers $\{k_1, k_2, \ldots, k_n\}$, such that $m \geq k_1 \geq k_2 \geq \ldots \geq k_n \geq 0$

7. Compute first homology group of torus with two punctures.