Quantum transport far from equilibrium: exactly solvable models and beyond

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Plan

• Introduction
• Quantum point contacts (QPCs): overview
• Exactly solvable model for a driven QPC
• Nonequilibrium phase transition in driven QPC
• Beyond exact solvability
• Summary and outlook
Research pipeline: from theory to applications
Schrödinger equation

Hamiltonian: describes the system
\[ i \frac{d}{dt} \Psi_t = \hat{H}_t \Psi_t \]

state vector: describes the state of the system

Hamiltonian that does not depend on time
stationary Schrödinger equation:
\[ H \Psi = E \Psi \]
equilibrium state vector = eigenstate
Far-from-equilibrium driven quantum many-body dynamics: complexity

**far-from-equilibrium driven quantum many-body dynamics**

- Differential equation with variable coefficients
- Vector of length $d$
- Matrix $d \times d$
- $d$ is exponential in system size
- E.g. $d = 2^N$ for $N$ qubits
- Store the state of:
  - ~40 qubits on laptop
  - ~75 qubits with the whole world data storage
Far-from-equilibrium driven quantum many-body dynamics: approaches

- approximate methods:
  - perturbation theory (small parameter required)
  - Floquet-Magnus theory (periodic driving, large frequencies)
  - various numerical techniques
  - ...

- exactly solvable models
Exactly solvable many-body models

Benefits:

• beautiful!

• bring an in-depth understanding of physical phenomena

• can be used to benchmark approximate techniques

Possible issue:

• can display non-generic behavior
Quantum point contact (QPC)

GaAs/AlGaAs heterostructure QPC

hBN–graphene QPC,

Driven QPC between two 1D conductors: a model

Maximally simplified model:

- noninteracting fermions
- conductors modelled as one-dimensional lattices
- time-dependent fields restricted to two edge sites

\[ j = 1 \quad 2 \quad \ldots \quad L-1 \quad L \quad L+1 \quad L+2 \quad \ldots \quad 2L \]
Driven QPC between two 1D conductors: a model

noninteracting fermions on a 1D lattice:

\[ H_t = H_L + H_R + V_t \]

\[ H_L = -\frac{1}{2} \sum_{j=1}^{L-1} (c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j), \quad H_R = -\frac{1}{2} \sum_{j=L+1}^{2L-1} (c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) \]

\[ V_t = -\frac{1}{2} \left( J_t c_L^\dagger c_{L+1} + J_t^* c_{L+1}^\dagger c_L + U_t^L c_L^\dagger c_L + U_t^R c_{L+1}^\dagger c_{L+1} \right) \]
Conformal QPC

\[ H_t = H_L + H_R + V_t \]

\[ V_t = -\frac{1}{2} \left( J_t c_L^\dagger c_{L+1} + J_t^* c_{L+1}^\dagger c_L + U_t^L c_L^\dagger c_L + U_t^R c_{L+1}^\dagger c_{L+1} \right) \]

\[ J_t = \sin \omega t, \quad U_t^L = -U_t^R = \cos \omega t \]
Floquet theory for periodically driven systems

\[ H_t = H_{t+\tau} \]

\[ \tau = \frac{2\pi}{\omega} \quad \text{– period} \]

stroboscopic times: \( t_n = n\tau, \quad n = 0, 1, 2, \ldots \)

stroboscopic dynamics: \( \Psi_{n\tau} = \mathcal{U}^n \Psi_0, \quad \mathcal{U} \equiv \text{Texp}\left( \int_0^t H_{t'} dt' \right) \)

\[ \mathcal{U} = e^{-i H_F \tau} \]

\( H_F \) – Floquet Hamiltonian
Floquet theory for periodically driven systems

\[ \Psi_{n\tau} = e^{-i n\tau H_F^{\text{conf}}} \Psi_0 \]

far-from-equilibrium quantum many-body dynamics

differential equations with \textit{variable} coefficients \quad \Rightarrow \quad differential equations with \textit{constant} coefficients

typically Floquet Hamiltonian is not known
Conformal QPC: exact Floquet Hamiltonian

\[ H_t^{\text{conf}} = e^{i\omega t \Sigma/2} H_0^{\text{conf}} e^{-i\omega t \Sigma/2} \]

\[ \Sigma = i \sum_{j=1}^{L} (c_j^d c_{2L+1-j} - \text{h.c.}) \]

\[ \Psi_t = e^{i\omega t \Sigma/2} e^{-i(H_0^{\text{conf}} + \omega \Sigma/2)t} \Psi_0 \]

\[ H_F^{\text{conf}} = H_0^{\text{conf}} + \frac{\omega}{2} \Sigma - \frac{\omega}{2} N \]
Conformal QPC: from driven to quench dynamics

\[ \Psi_{n\tau} = e^{-i n\tau H_F^{\text{conf}}} \Psi_0 \]

dynamics of noninteracting fermions with \textit{time-independent} Hamiltonian

amenable to analytical treatment!
Conformal QPC: heating rate and current

$$\overline{\mathcal{W}} = \int \frac{dE}{2\pi \tau} [\rho_L(E) + \rho_R(E)] \Gamma_\omega(E),$$

- heating rate

$$\overline{\mathcal{J}} = \int \frac{dE}{2\pi} [\rho_L(E) - \rho_R(E)] T_\omega(E),$$

- current

![Graphs showing the behavior of $T_\omega(E)$ and $\Gamma_\omega(E)$ for different values of $\omega$.](image)
Conformal QPC: heating function and transmission coefficient

\[ T_{\omega}(E) = \text{Re} \left\{ (1 - E^2) \left[ 1 - \left( \frac{\sqrt{(E - \omega)^2 - 1} + \sqrt{(E + \omega)^2 - 1}}{2\omega} \right)^2 \right] \right. \]

\[ + \frac{\sqrt{1 - E^2}}{2\omega^2} \left[ \sqrt{1 - (E - \omega)^2(E^2 + E\omega - 1)} + \sqrt{1 - (E + \omega)^2(E^2 - E\omega - 1)} \right] \right\}, \]

\[ \Gamma_{\omega}(E) = 2\pi \frac{\sqrt{1 - E^2}}{\omega^2} \text{Re} \left\{ \sqrt{1 - (E - \omega)^2(E^2 - E\omega - 1)} - \sqrt{1 - (E + \omega)^2(E^2 + E\omega - 1)} \right\} \]

\[ + 2\pi \frac{1 - E^2}{\omega^2} \left\{ [(E + \omega)^2 - 1]\theta(1 - \omega - E) - [(E - \omega)^2 - 1]\theta(E - \omega + 1) \right\}. \]
Conformal QPC: no heating above critical frequency

\[ \overline{w} = \int \frac{dE}{2\pi \tau} [\rho_L(E) + \rho_R(E)] \Gamma_\omega(E). \]

\[ \Gamma_\omega(E) = 0 \quad \text{for} \quad \omega \geq \omega_c = 2 \]
Other QPCs: heating above $\omega_c$ still absent

exploring non-exactly-solvable QPCs numerically:
Tunneling QPC: no current above $\omega_c$

Tunneling QPC:

\[ J_t = \sin \omega t \]

\[ U^L_t = U^R_t = 0 \]

Numerical observation:

\[ \overline{J} = 0 \quad \text{for} \quad \omega > \omega_c \]
Beyond conformal QPC: no current above $\omega_c$

**tunneling QPC:**

$$J_t = \sin \omega t$$

$$U_t^L = U_t^R = 0$$

**numerical observation:**

$$\overline{J} = 0 \quad \text{for} \quad \omega > \omega_c$$
No current above $\omega_c$: when and why?

In general, the question is open

Numerics suggests that the following conditions are sufficient (but not necessary!):

- $\int_0^\tau J_t \, dt = 0$
- $J_t$ is real
- $\partial_t U_t = 0$

No deep understanding of the effect, calls for further studies!
Approximate techniques: Floquet-Magnus expansion

Expansion at large frequencies:

\[ H_F = \sum_{n=0}^{\infty} \omega^{-n} M^{(n)} \]

\[
M^{(0)} = \tau^{-1} \int_0^\tau H_{t_1} \, dt_1, \\
M^{(1)} = \frac{i \omega}{2} \int_0^\tau dt_1 \int_0^{t_1} dt_2 \left[ H_{t_1}, H_{t_2} \right], \\
M^{(2)} = \frac{\omega^2}{6} \int_0^\tau dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 \left( \left[ H_{t_1}, \left[ H_{t_2}, H_{t_3} \right] \right] + \left[ H_{t_3}, \left[ H_{t_2}, H_{t_1} \right] \right] \right).
\]

Requires \( O(L) \) terms to get a true Floquet Hamiltonian:
Approximate techniques: perturbation theory

Small $V_t$ can be treated as a perturbation

$$\mathcal{W}^{(1)} = \int \frac{dE}{2\pi \tau} \left( \frac{|J|^2 + |U_L|^2}{2|J|^2} \rho_L + \frac{|J|^2 + |U_R|^2}{2|J|^2} \rho_R \right) \Gamma_{\omega}^{(1)}$$

$$\mathcal{J}^{(1)} = \int \frac{dE}{2\pi} (\rho_L - \rho_R) T_{\omega}^{(1)}$$

$$\Gamma_{\omega}^{(1)} = 4\pi |J|^2 \sqrt{1 - E^2} \text{Re}[\sqrt{1 - (E + \omega)^2} - \sqrt{1 - (E - \omega)^2}],$$

$$T_{\omega}^{(1)} = \sqrt{1 - E^2} \left\{ |J|^2 \text{Re}[\sqrt{1 - (E + \omega)^2}ight.$$  
$$+ \sqrt{1 - (E - \omega)^2} + 4|J|^2 \sqrt{1 - E^2} \right\},$$
Approximate techniques: perturbation theory

\[ \overline{\mathcal{W}}^{(1)} = 0 \]
\[ \overline{\mathcal{J}}^{(1)} = 0 \]

when \( \omega > \omega_c \)

for arbitrary QPC

\[ J_t = 0.3 \sin \omega t, \quad U_t^L = -U_t^R = 0.3 \cos \omega t \]

Does not shed light on conditions for current vanishing
Interacting fermions

Tentitative conclusions (with Ivan Dudinets):

- Interaction only within QPC: all the effects remain

- Interaction in the bulk: heating rate and current are (at least) strongly suppressed above the critical frequency

Work in progress!
Outlook: possible applications

• Absence of heating – (almost) always welcome!

• Switching the current on and off by varying the frequency: frequency-controlled quantum switch!
Summary

• exactly solvable model for a driven QPC studied

• nonequilibrium phase transition above a critical frequency discovered

• phase transition is there for other (non-exactly-solvable) models

• heating rate (always) and current (sometimes) vanish above the critical frequency

• a frequency-controlled quantum switch can be anticipated
Funding

Project  Statistical behavior of thermally isolated many-body quantum systems

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post scriptum

We also use conformal QPC to benchmark quantum speed limits and adiabatic conditions for many-body systems

Project *Quantum adiabaticity in many-body systems*
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Thank you for your attention!
Miscellaneous