Quantum transport far from equilibrium: exactly solvable models and beyond

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Plan

- Introduction
- Quantum point contacts (QPCs): overview
- Exactly solvable model for a driven QPC
- Nonequilibrium phase transition in driven QPC
- Beyond exact solvability
- Summary and outlook

Research pipeline: from theory to applications

Schrödinger equation

Hamiltonian: describes the system

Schrödinger equation: $$i\frac{d}{dt}\Psi_t = H_t\,\Psi_t$$

state vector: describes the state of the system

Hamiltonian that does not depend on time

stationary Schrödinger equation: $$H\,\Psi = E\,\Psi$$

equilibrium state vector = eigenstate

Far-from-equilibrium driven quantum many-body dynamics: complexity

far-from-equilibrium driven quantum many-body dynamics

d is exponential in system size

vector of length d

$$i\frac{d}{dt}\Psi_t = H_t\,\Psi_t \qquad H\,\Psi = E\,\Psi$$

matrix $d \times d$

e.g. $d = 2^N$ for N qubits

store the state of:

~ 40 qubits on laptop

~ 75 qubits with the whole world data storage

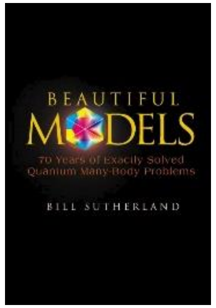
differential equation with variable coefficients

Far-from-equilibrium driven quantum many-body dynamics: approaches

- approximate methods:
  - perturbation theory (small parameter required)
  - Floquet-Magnus theory (periodic driving, large frequencies)
  - various numerical techniques
  - ...
- exactly solvable models

Exactly solvable many-body models

Benefits:

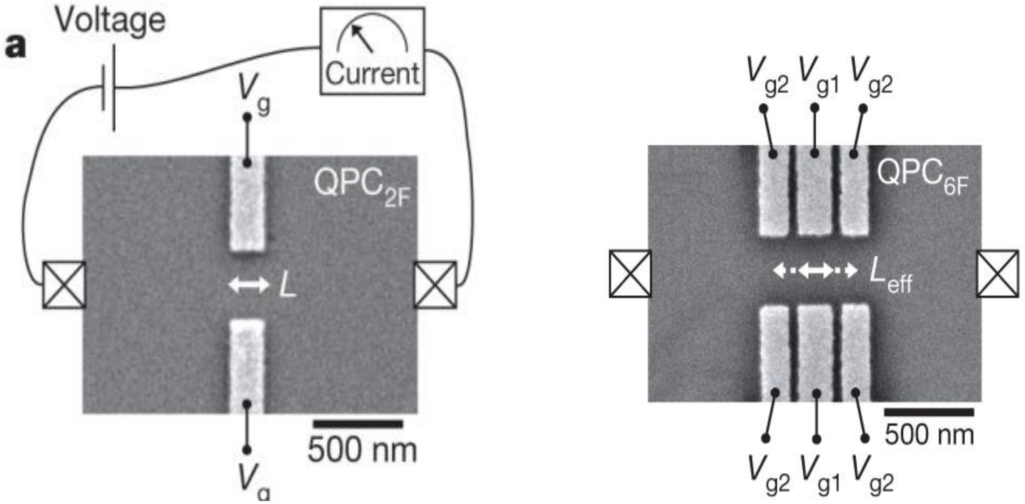


- beautiful!
- bring an in-depth understanding of physical phenomena
- can be used to benchmark approximate techniques

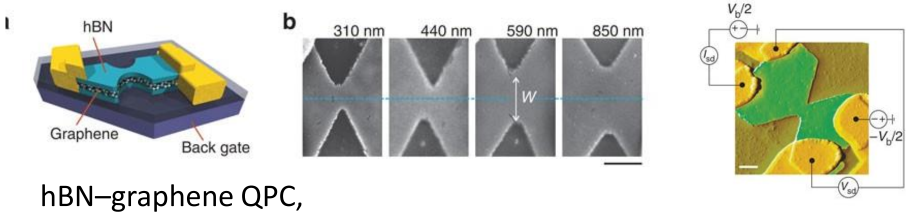
Possible issue:

- can display non-generic behavior

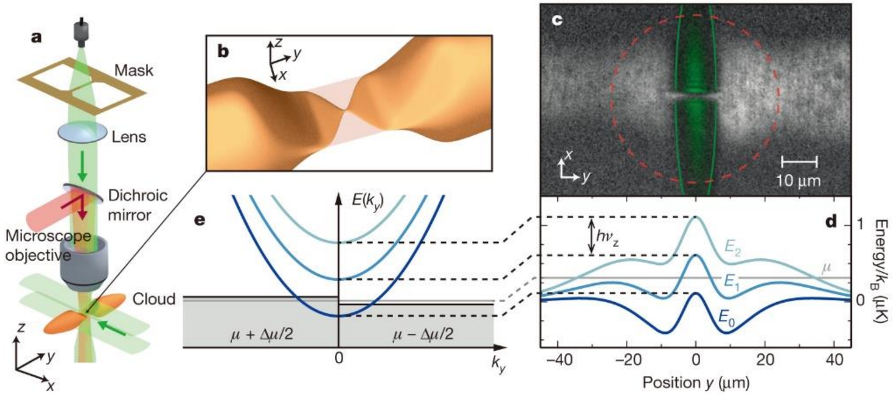
Quantum point contact (QPC)



GaAs/AlGaAs heterostructure QPC
MJ Iqbal *et al. Nature* **501,** 79–83 (2013)



hBN–graphene QPC,
B. Terrés et al *Nat Commun* **7,** 11528 (2016)

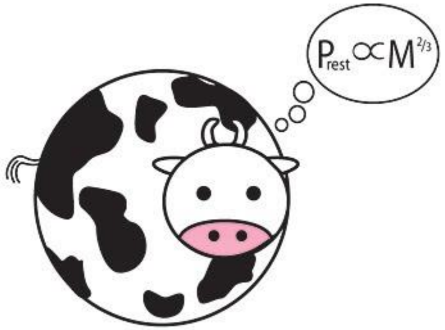


Atomic QPC. S Krinner *et al. Nature* **517**, 64-67 (2015)

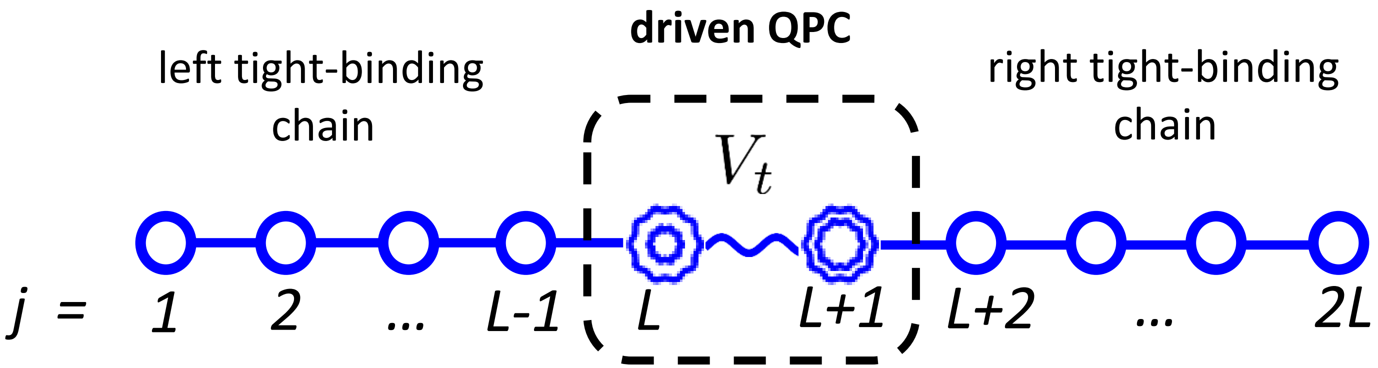
Driven QPC between two 1D conductors: a model

Maximally simplified model:

- noninteracting fermions
- conductors modelled as one-dimensional lattices
- time-dependent fields restricted to two edge sites



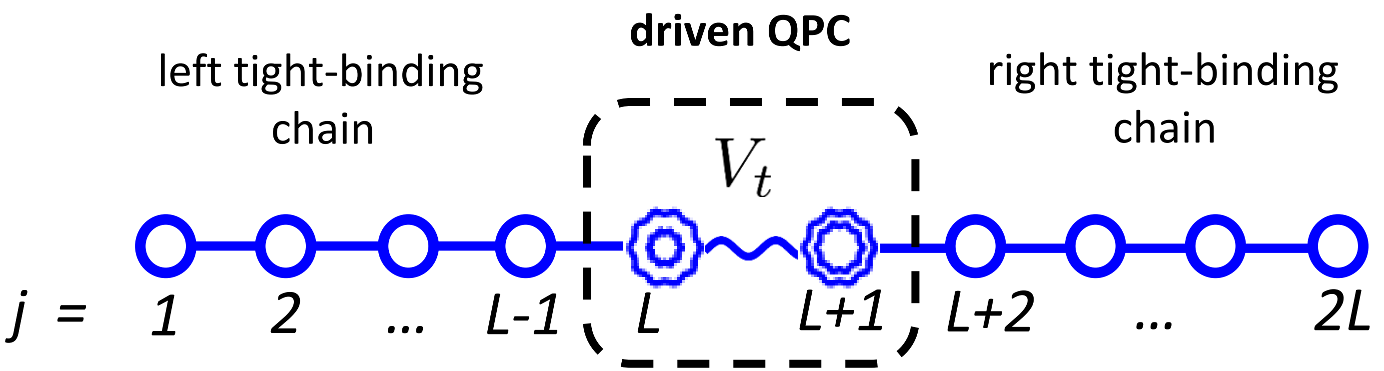
from http://psychsciencenotes.blogspot.com/



Driven QPC between two 1D conductors: a model

noninteracting fermions on a 1D lattice:

$$H_t = H_L + H_R + V_t$$

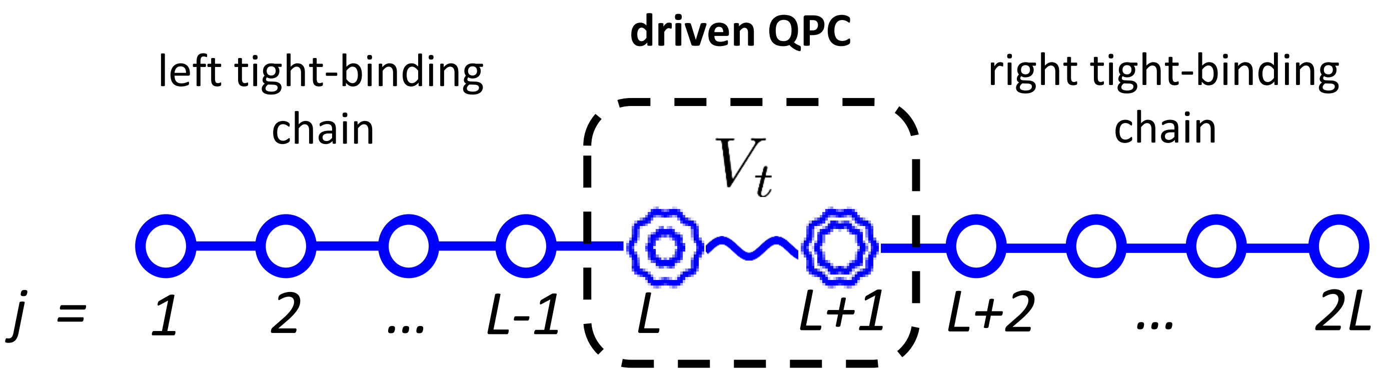


$$H_L = -\frac{1}{2}\sum_{j=1}^{L-1}(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j), \qquad H_R = -\frac{1}{2}\sum_{j=L+1}^{2L-1}(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j)$$

$$V_t = -\frac{1}{2}\left(J_t\, c_L^\dagger\, c_{L+1} + J_t^*\, c_{L+1}^\dagger\, c_L + U_t^L\, c_L^\dagger\, c_L + U_t^R\, c_{L+1}^\dagger\, c_{L+1}\right)$$

Conformal QPC

$$H_t = H_L + H_R + V_t$$



$$V_t = -\frac{1}{2}\left(J_t\, c_L^\dagger\, c_{L+1} + J_t^*\, c_{L+1}^\dagger\, c_L + U_t^L\, c_L^\dagger\, c_L + U_t^R\, c_{L+1}^\dagger\, c_{L+1} \right)$$

$$\boxed{J_t = \sin\omega t, \quad U_t^L = -U_t^R = \cos\omega t}$$

Floquet theory for periodically driven systems

$$H_t = H_{t+\tau}$$

$\tau = 2\pi/\omega$ – period

stroboscopic times: $t_n = n\tau, \quad n = 0, 1, 2, \ldots$

stroboscopic dynamics: $\Psi_{n\tau} = \mathcal{U}^n \Psi_0, \qquad \mathcal{U} \equiv \mathrm{Texp}\Big(\int_0^t H_{t'} dt' \Big)$

$$\mathcal{U} = e^{-i\, H_{\mathrm{F}} \tau}$$

H_{F} – Floquet Hamiltonian

Floquet theory for periodically driven systems

$$\Psi_{n\tau} = e^{-i\,n\tau\,H_{\mathrm{F}}^{\mathrm{conf}}}\,\Psi_0$$

far-from-equilibrium **quantum many-body dynamics**

differential equations
with **variable** coefficients

$\Rightarrow$

differential equations
with **constant** coefficients

typically Floquet Hamiltonian is not known

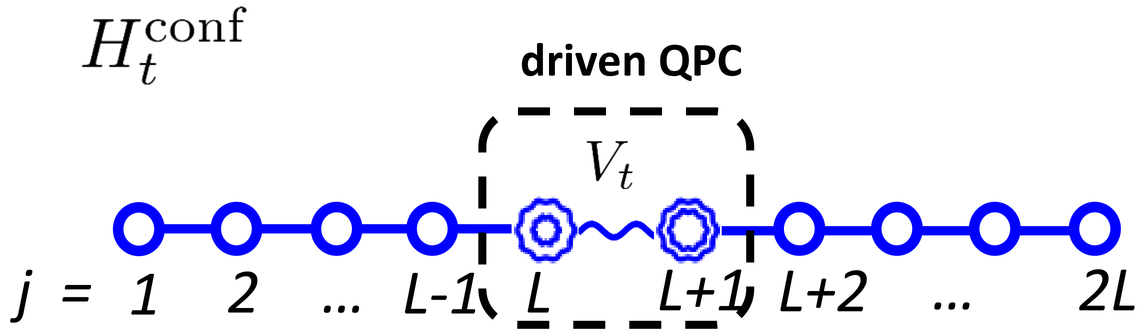
Conformal QPC: exact Floquet Hamiltonian

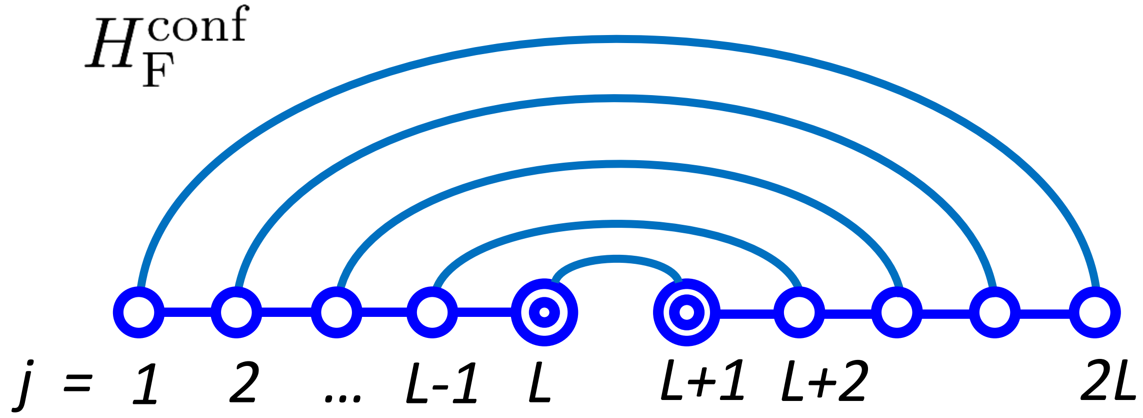
$$H_t^{\mathrm{conf}} = e^{i\omega t\Sigma/2} H_0^{\mathrm{conf}}\, e^{-i\omega t\Sigma/2}$$

$$\Sigma = i \sum_{j=1}^{L} (c_j^\dagger c_{2L+1-j} - \mathrm{h.c.})$$

$$\Psi_t = e^{i\omega t\Sigma/2} e^{-i\left(H_0^{\mathrm{conf}} + \omega\Sigma/2\right)t}\,\Psi_0$$

$$\boxed{H_{\mathrm{F}}^{\mathrm{conf}} = H_0^{\mathrm{conf}} + \frac{\omega}{2}\Sigma - \frac{\omega}{2}N}$$





Conformal QPC: from driven to quench dynamics

$$\Psi_{n\tau} = e^{-i\,n\tau\,H_{\mathrm{F}}^{\mathrm{conf}}}\,\Psi_0$$

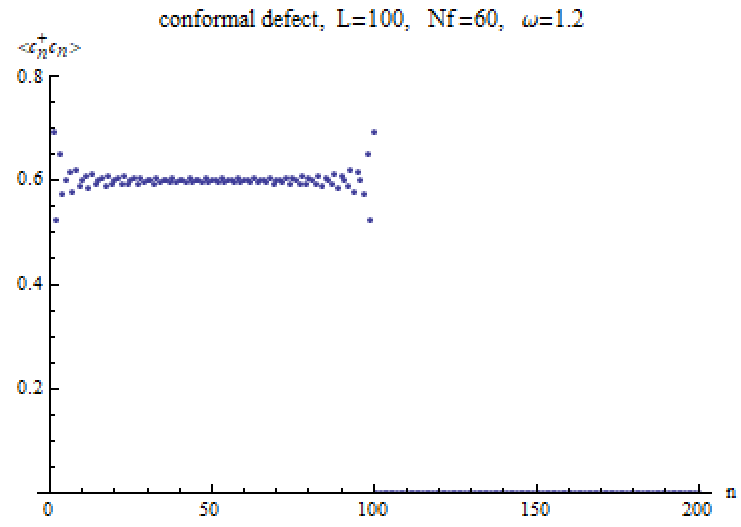
dynamics of noninteracting fermions with ***time-independent*** Hamiltonian

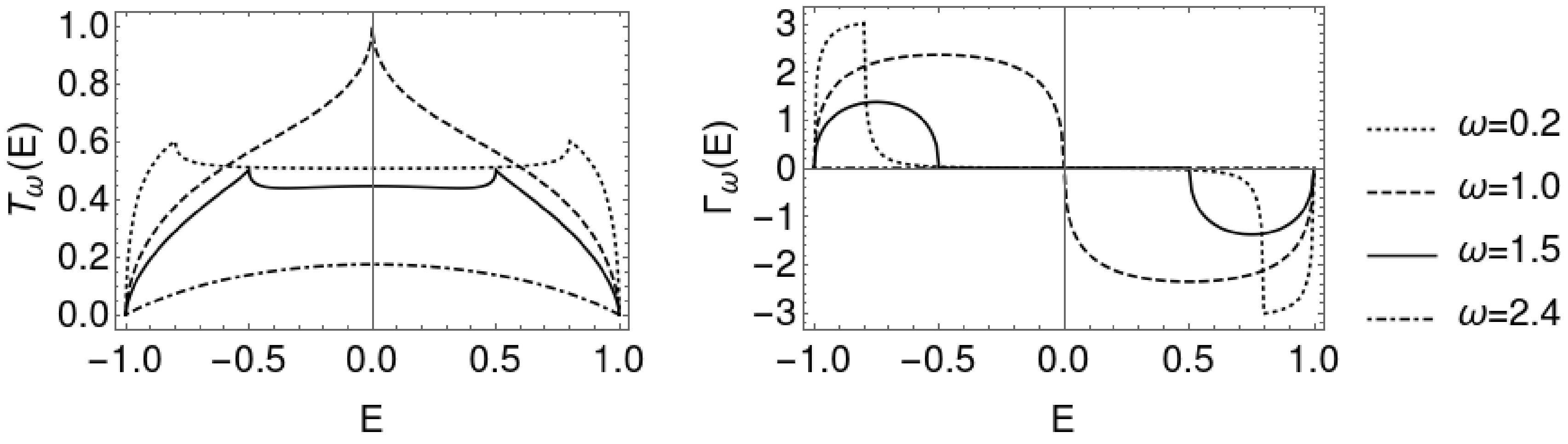
amenable to analytical treatment!

Conformal QPC: heating rate and current

$$\overline{\mathscr{W}} = \int \frac{dE}{2\pi\tau}[\rho_L(E)+\rho_R(E)]\Gamma_\omega(E),$$ – heating rate

$$\overline{\mathcal{J}} = \int \frac{dE}{2\pi}[\rho_L(E)-\rho_R(E)]T_\omega(E).$$ – current





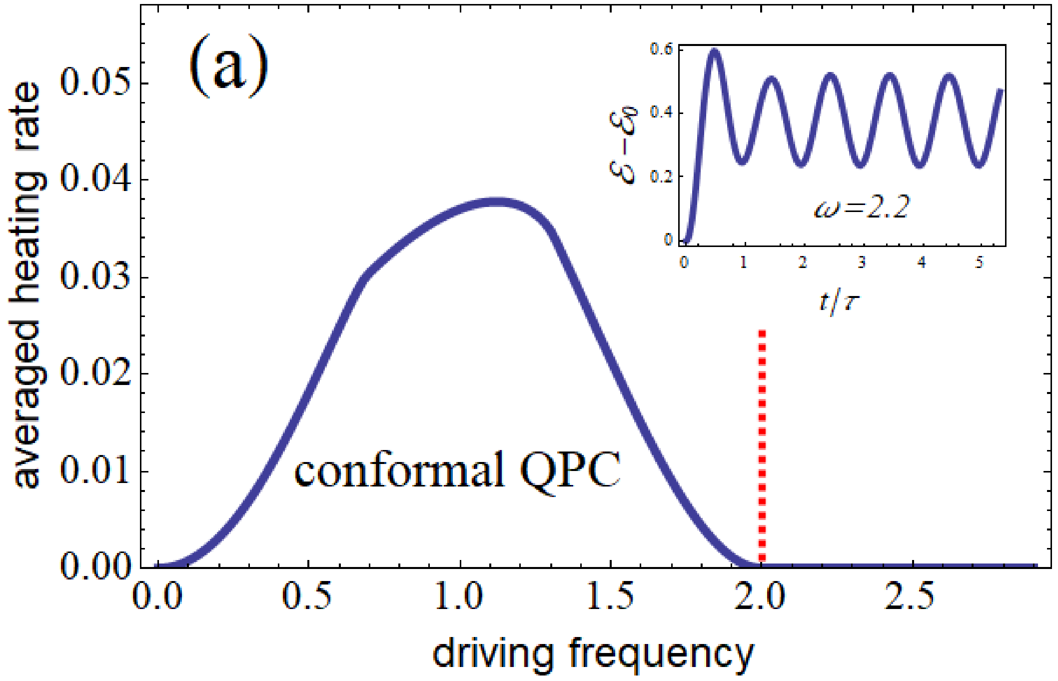
Conformal QPC: heating function and transmission coefficient

$$T_\omega(E) = \mathrm{Re}\left\{(1-E^2)\left[1-\left(\frac{\sqrt{(E-\omega)^2-1}+\sqrt{(E+\omega)^2-1}}{2\omega}\right)^2\right]\right.$$
$$\left.+\frac{\sqrt{1-E^2}}{2\omega^2}\left[\sqrt{1-(E-\omega)^2}(E^2+E\omega-1)+\sqrt{1-(E+\omega)^2}(E^2-E\omega-1)\right]\right\},$$

$$\Gamma_\omega(E) = 2\pi\frac{\sqrt{1-E^2}}{\omega^2}\mathrm{Re}\{[\sqrt{1-(E-\omega)^2}(E^2-E\omega-1)-\sqrt{1-(E+\omega)^2}(E^2+E\omega-1)]\}$$
$$+2\pi\frac{1-E^2}{\omega^2}\{[(E+\omega)^2-1]\theta(1-\omega-E)-[(E-\omega)^2-1]\theta(E-\omega+1)\}.$$

Conformal QPC: no heating above critical frequency

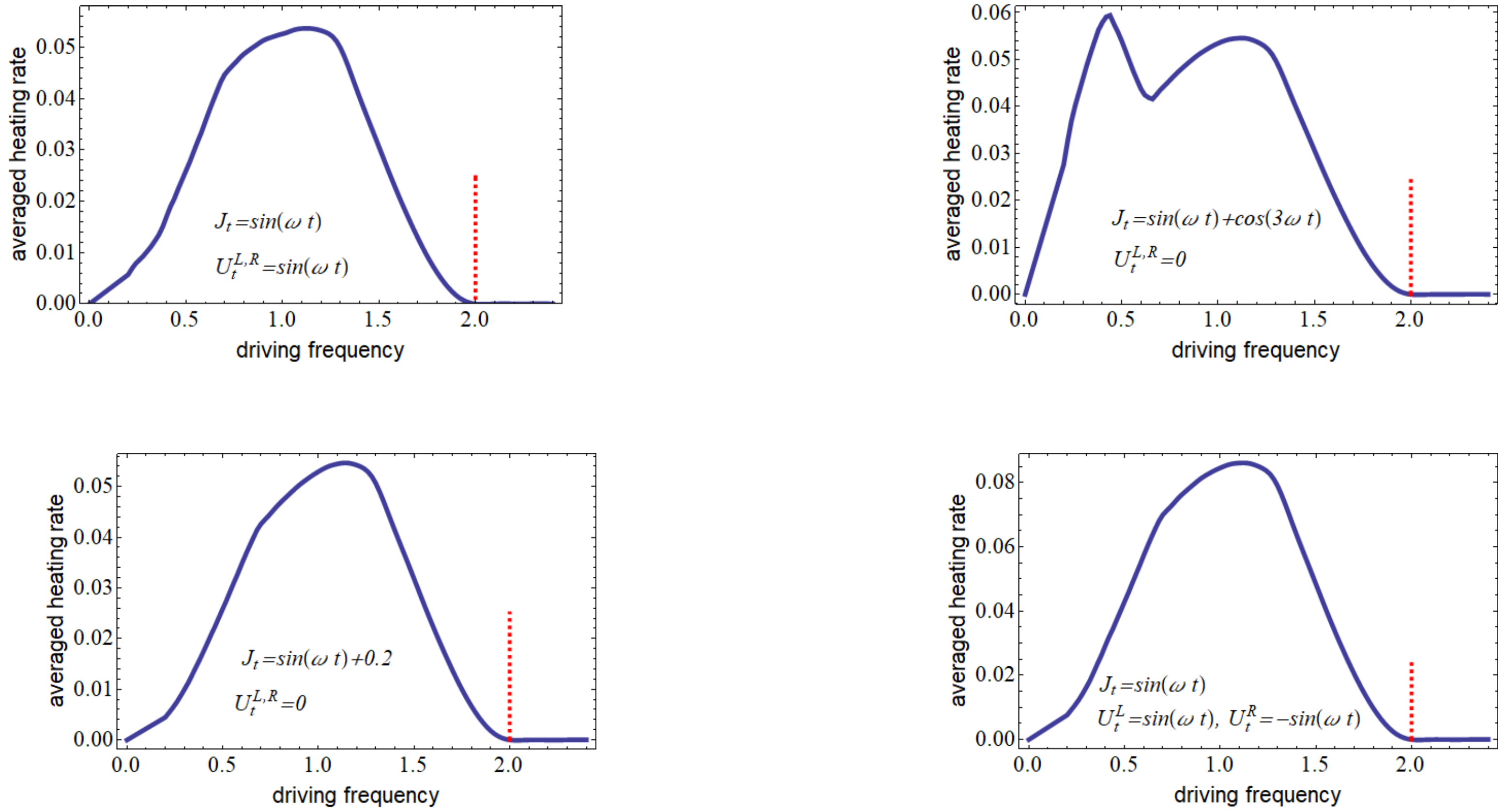
$$\overline{\mathscr{W}} = \int \frac{dE}{2\pi\tau}[\rho_L(E)+\rho_R(E)]\Gamma_\omega(E),$$



$$\boxed{\Gamma_\omega(E) = 0 \qquad \text{for} \qquad \omega \geq \omega_c = 2}$$

Other QPCs: heating above ω_c still absent

exploring non-exactly-solvable QPCs numerically:



Tunneling QPC: no current above ω_c

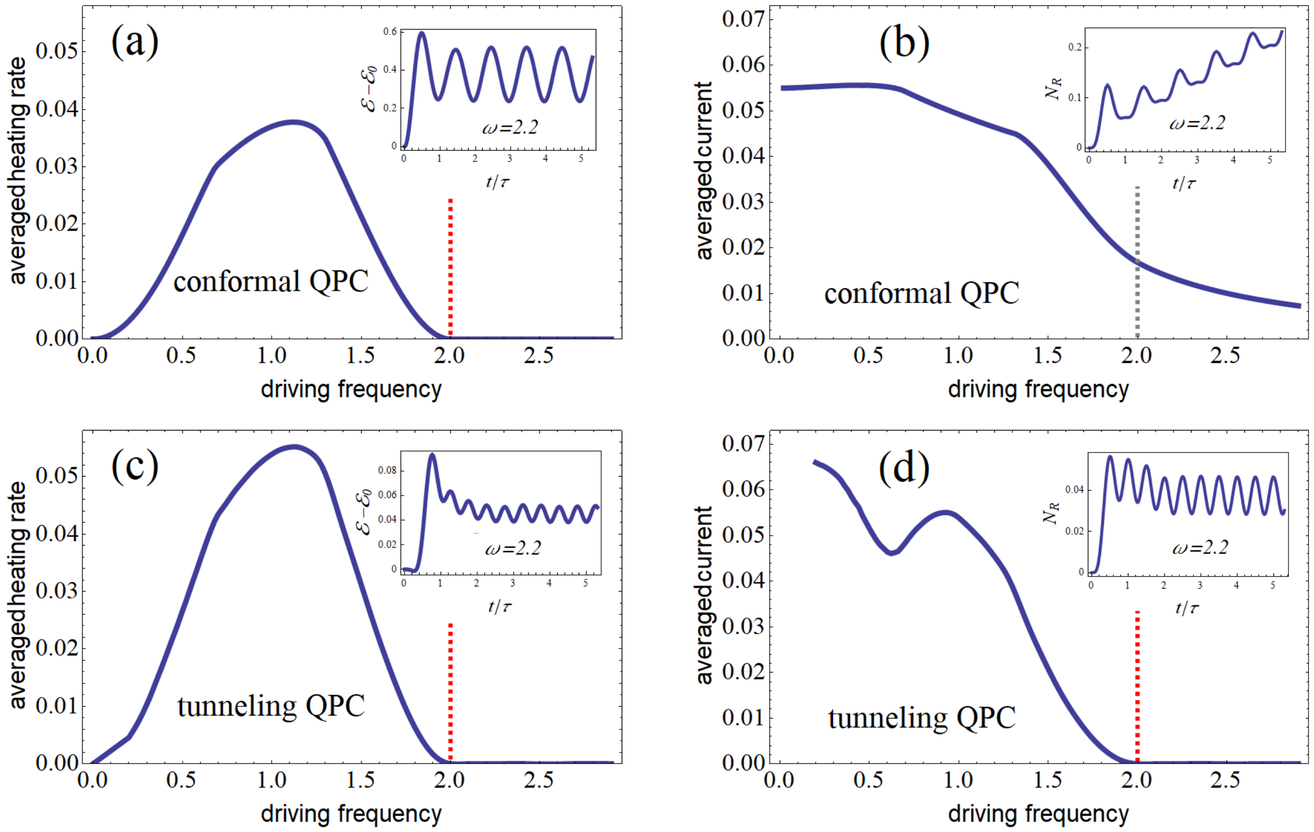
tunneling QPC:

$$J_t = \sin \omega t$$

$$U_t^L = U_t^R = 0$$

numerical observation:

$$\boxed{\overline{\mathcal{J}} = 0 \qquad \text{for} \qquad \omega > \omega_c}$$



Beyond conformal QPC: no current above ω_c

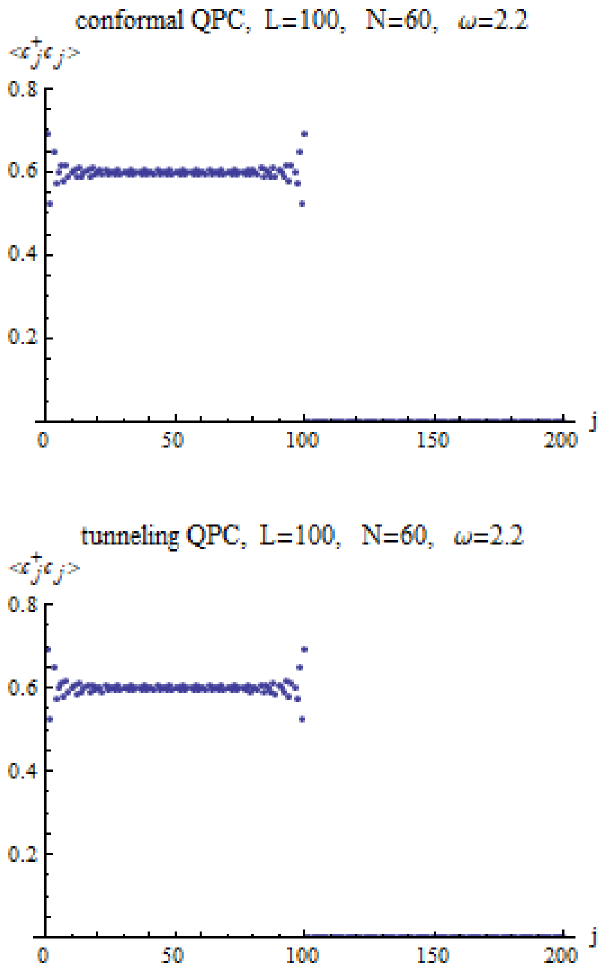
tunneling QPC:

$$J_t = \sin \omega t$$

$$U_t^L = U_t^R = 0$$

numerical observation:

$$\boxed{\overline{\mathcal{J}} = 0 \qquad \text{for} \qquad \omega > \omega_c}$$



No current above ω_c: when and why?

In general, the question is open

Numerics suggests that the following conditions are sufficient (but not necessary!):

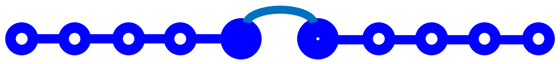
- $\int_0^\tau J_t \, dt = 0$
- J_t is real
- $\partial_t U_t = 0$

No deep understanding of the effect, calls for further studies!

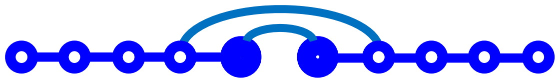
Approximate techniques: Floquet-Magnus expansion

expansion at large frequencies: $H_{\mathrm{F}} = \sum_{n=0}^{\infty} \omega^{-n} M^{(n)}$

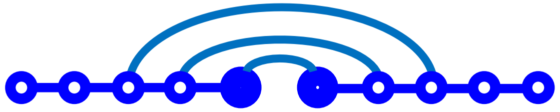
$$M^{(0)} = \tau^{-1} \int_0^{\tau} H_{t_1} dt_1,$$



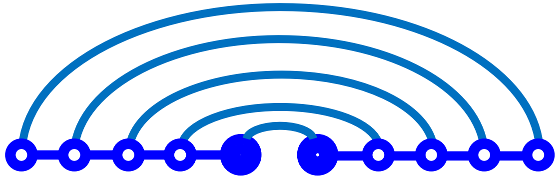
$$M^{(1)} = \frac{i\omega}{2} \int_0^{\tau} dt_1 \int_0^{t_1} dt_2 \, [H_{t_1}, H_{t_2}],$$



$$M^{(2)} = \frac{\omega^2}{6} \int_0^{\tau} dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 \left([H_{t_1}, [H_{t_2}, H_{t_3}] + [H_{t_3}, [H_{t_2}, H_{t_1}], \right),$$



requires *O(L)* terms to get a true Floquet Hamiltonian:



Approximate techniques: perturbation theory

small V_t can be treated as a perturbation

$$\overline{\mathscr{W}}^{(1)} = \int \frac{dE}{2\pi\tau}\left(\frac{|J|^2+|U_L|^2}{2|J|^2}\rho_L + \frac{|J|^2+|U_R|^2}{2|J|^2}\rho_R\right)\Gamma^{(1)}_\omega$$

$$\overline{\mathscr{J}}^{(1)} = \int \frac{dE}{2\pi}(\rho_L - \rho_R)T^{(1)}_\omega$$

$$\Gamma^{(1)}_\omega = 4\pi|J|^2\sqrt{1-E^2}\,\mathrm{Re}[\sqrt{1-(E+\omega)^2} - \sqrt{1-(E-\omega)^2}],$$

$$T^{(1)}_\omega = \sqrt{1-E^2}\,\{|J|^2\,\mathrm{Re}[\sqrt{1-(E+\omega)^2} + \sqrt{1-(E-\omega)^2}] + 4|\bar{J}|^2\sqrt{1-E^2}\},$$

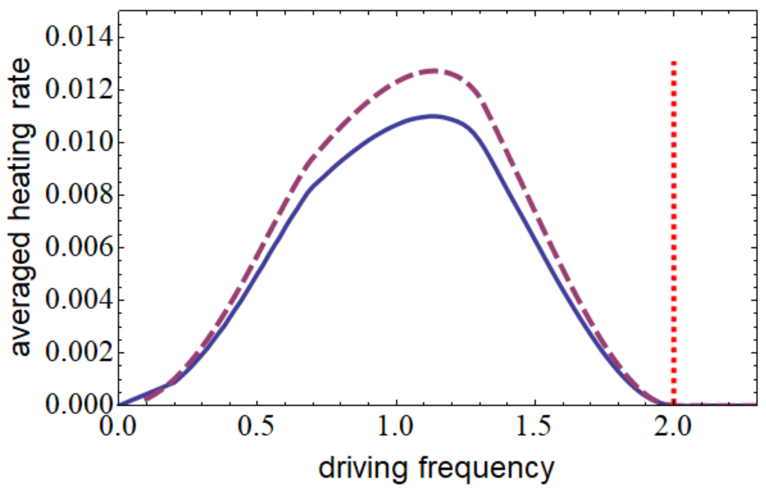
Approximate techniques: perturbation theory

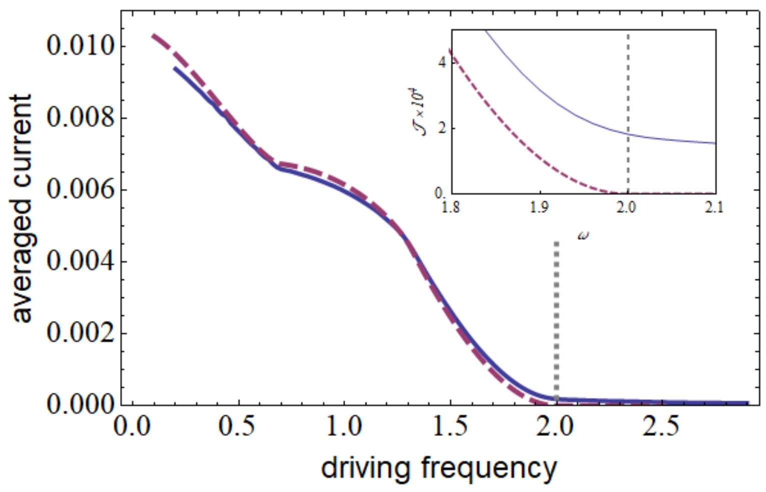
$$\overline{\mathscr{W}}^{(1)} = 0$$

$$\overline{\mathcal{J}}^{(1)} = 0$$

when $\omega > \omega_c$

for ***arbitrary*** QPC





$$J_t = 0.3 \sin \omega t, \quad U_t^L = -U_t^R = 0.3 \cos \omega t$$

Does not shed light on conditions for current vanishing

Interacting fermions

Tentitative conclusions (with Ivan Dudinets):

- Interaction only within QPC: all the effects remain
- Interaction in the bulk: heating rate and current are (at least) strongly suppressed above the critical frequency

Work in progress!

Outlook: possible applications

- Absence of heating – (almost) always welcome!
- Switching the current on and off by varying the frequency:

frequency-controlled quantum switch!

Summary

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Letter

Nonequilibrium phase transition in transport through a driven quantum point contact

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We study the transport of noninteracting fermions through a periodically driven quantum point contact (QPC) connecting two tight-binding chains. Initially, each chain is prepared in its own equilibrium state, generally with a bias in chemical potentials and temperatures. We examine the heating rate (or, alternatively, energy increase per cycle) in the nonequilibrium time-periodic steady state established after initial transient dynamics. We find that the heating rate vanishes identically when the driving frequency exceeds the bandwidth of the chain. We first establish this fact for a particular type of QPCs where the heating rate can be calculated analytically. Then we verify numerically that this nonequilibrium phase transition is present for a generic QPC. Finally, we derive this effect perturbatively in leading order for cases when the QPC Hamiltonian can be considered a small perturbation. Strikingly, we discover that for certain QPCs the current averaged over the driving cycle also vanishes above the critical frequency, despite a persistent bias. This shows that a driven QPC can act as a frequency-controlled quantum switch.

- exactly solvable model for a driven QPC studied
- nonequilibrium phase transition above a critical frequency discovered
- phase transition is there for other (non-exactly-solvable) models
- heating rate (always) and current (sometimes) vanish above the critical frequency
- a frequency-controlled quantum switch can be anticipated

Funding

Project *Statistical behavior of thermally isolated many-body quantum systems*

Russian Science Foundation grant No 17-12-01587

post scriptum

We also use conformal QPC to benchmark *quantum speed limits* and *adiabatic conditions* for many-body systems

Project ***Quantum adiabaticity in many-body systems***
Russian Science Foundation grant No 17-71-20158

Thank you for your attention!

Miscellaneous