Quantum speed limit for thermal states

Nikolay Il’in, Oleg Lychkovskiy

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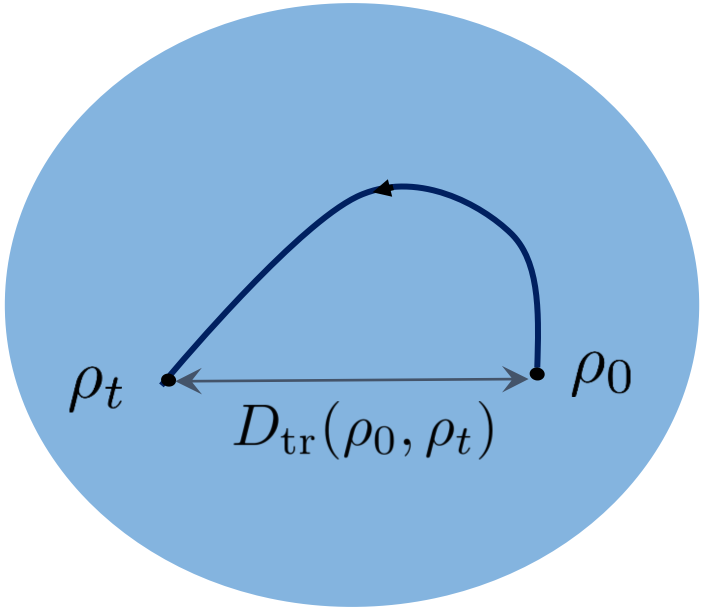
What is quantum speed limit (QSL)?

closed quantum system (in general, in a mixed state):

$$i\partial_t \rho_t = [H, \rho_t]$$

trace distance: $D_{\mathrm{tr}}(\rho_1, \rho_2) \equiv (1/2)\,\mathrm{tr}|\rho_2 - \rho_1|$



Mandelstam-Tamm QSL (1945):

$$D_{\mathrm{tr}}(\rho_0, \rho_t) \le \Delta E\, t \qquad \Delta E \equiv \sqrt{\langle H^2 \rangle - \langle H \rangle^2}, \qquad \langle A \rangle \equiv \mathrm{tr}\, \rho_0 A$$

out-of-equilibrium evolution

observable in the initial (often equilibrium) state

A remark on good and bad distances

In the many-body setting not all distances are equally meaningful!

Good distances (faithfully measure the distinguishability of states):

- trace distance
- Bures distance
- Hellinger distance

Bad distance:

- Hilbert-Schmidt distance – can nearly vanish for orthogonal states

see more in Markham *et al*, Phys. Rev. A 77, 042111 (2008)

Zoo of quantum speed limits

Mandelstam, Tamm (MT, 1945): $D_{\mathrm{tr}}(\rho_0, \rho_t) \leq \Delta E\, t$ $\quad \Delta E \equiv \sqrt{\langle H^2 \rangle - \langle H \rangle^2}$

Margolus, Levitin (ML, 1998): $D_{\mathrm{tr}}(\rho_0, \rho_t) \leq \sqrt{2\, \overline{E}\, t}$ $\quad \overline{E} \equiv \langle H \rangle - E_{\mathrm{gs}}$

Mondal, Datta, Sazim (MDS, 2016): $D_{\mathrm{tr}}(\rho_0, \rho_t) \leq \delta E\, t$

$$\delta E \equiv \sqrt{-\mathrm{tr}\left[\sqrt{\rho_0}, H\right]^2}$$

Thermal initial state

$$\rho_0 = e^{-\beta H_0}/Z_0, \quad Z_0 = \mathrm{tr} e^{-\beta H_0}$$

initial Hamiltonian

$$H = H_0 + V$$

perturbation

V is not assumed to be small

$$i\partial_t \rho_t = [H_0 + V, \rho_t]$$

MT and ML QSLs for many-body thermal states

Mandelstam-Tamm: $D_{\mathrm{tr}}(\rho_0, \rho_t) \le \Delta E\, t \quad \Delta E \equiv \sqrt{\langle (H_0+V)^2\rangle - \langle H_0 + V\rangle^2}$

$\Delta E \sim \sqrt{N}$ in the thermodynamic limit (where N is the system size)

Margolus-Levitin: $D_{\mathrm{tr}}(\rho_0, \rho_t) \le \sqrt{2\,\overline{E}\, t} \qquad \overline{E} \equiv \langle H_0 + V\rangle - E_{\mathrm{gs}}$

$\sqrt{\overline{E}} \sim \sqrt{N}$ in the thermodynamic limit

Quantum speed limit for a thermal initial state

$$\rho_0 = e^{-\beta H_0}/Z_0, \quad Z_0 = \mathrm{tr} e^{-\beta H_0}$$

$$i\partial_t \rho_t = [H_0 + V, \rho_t]$$

Thermal QSL: $$D_{\mathrm{tr}}(\rho_0, \rho_t) \leq \sqrt{\beta t} \sqrt[4]{-2 \langle [H_0, V]^2 \rangle_\beta}$$

main result

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$$\langle A \rangle_\beta \equiv \mathrm{tr}\, \rho_0 A$$

T-QSL *vs* generic QSLs: infinite temperature

Finite Hilbert space with dimension d, $\beta = 0$

$$\left.\begin{aligned} & i\partial_t \rho_t = [H_0 + V, \rho_t] \\ & \rho_0 = d^{-1}\,\mathbb{1} \end{aligned}\right\} \Longrightarrow \begin{aligned} & \rho_t = \rho_0 = d^{-1}\,\mathbb{1} \\ & D_{\mathrm{tr}}(\rho_0, \rho_t) = 0 \end{aligned}$$

T-QSL *vs* generic QSLs: infinite temperature

$\beta = 0$ $\qquad \rho_t = \rho_0 = d^{-1}\,\mathbb{1}$ $\qquad D_{\mathrm{tr}}(\rho_0, \rho_t) = 0$

MT $\quad D_{\mathrm{tr}}(\rho_0, \rho_t) \leq \Delta E\, t$

$$\Delta E \equiv \sqrt{\langle H^2\rangle - \langle H\rangle^2} \sim \sqrt{N}$$

✗

MDS $\quad D_{\mathrm{tr}}(\rho_0, \rho_t) \leq \delta E\, t$

$$\delta E \equiv \sqrt{-\mathrm{tr}\,[\sqrt{\rho_0}, V]^2} = 0$$

✓

ML $\quad D_{\mathrm{tr}}(\rho_0, \rho_t) \leq \sqrt{2\,\overline{E}\,t}$

$$\sqrt{\overline{E}} \equiv \sqrt{\langle H\rangle - E_{\mathrm{gs}}} \sim \sqrt{N}$$

✗

thermal

$$D_{\mathrm{tr}}(\rho_0, \rho_t) \leq \sqrt{\underline{\beta} t}\,\sqrt[4]{-2\,\langle [H_0, V]^2\rangle_\beta}$$

✓

T-QSL *vs* generic QSLs: trivial perturbation

trivial perturbation: $[V, H_0] = 0$

$$\left.\begin{aligned} i\partial_t \rho_t &= [H_0 + V, \rho_t] \\ \rho_0 &= e^{-\beta H_0}/Z_0 \end{aligned}\right\} \Longrightarrow \quad \begin{aligned} \rho_t &= \rho_0 \\ D_{\mathrm{tr}}(\rho_0, \rho_t) &= 0 \end{aligned}$$

T-QSL *vs* generic QSLs: trivial perturbation

$$[V, H_0] = 0 \qquad \rho_t = \rho_0 \qquad D_{\mathrm{tr}}(\rho_0, \rho_t) = 0$$

MT $\quad D_{\mathrm{tr}}(\rho_0, \rho_t) \leq \Delta E\, t$

$$\Delta E \equiv \sqrt{\langle H^2 \rangle - \langle H \rangle^2} \sim \sqrt{N}$$

✗

MDS $\quad D_{\mathrm{tr}}(\rho_0, \rho_t) \leq \delta E\, t$

$$\delta E \equiv \sqrt{-\mathrm{tr}\left[\sqrt{\rho_0}, V\right]^2} = 0$$

✓

ML $\quad D_{\mathrm{tr}}(\rho_0, \rho_t) \leq \sqrt{2\,\overline{E}}\, t$

$$\sqrt{\overline{E}} \equiv \sqrt{\langle H \rangle - E_{\mathrm{gs}}} \sim \sqrt{N}$$

✗

thermal

$$D_{\mathrm{tr}}(\rho_0, \rho_t) \leq \sqrt{\beta t}\, \sqrt[4]{-2 \left\langle [H_0, V]^2 \right\rangle_\beta}$$

✓

MDS QSL: computability issue

MDS $\quad D_{\mathrm{tr}}(\rho_0, \rho_t) \leq \delta E\, t$

$$\delta E^2 = -\mathrm{tr}\,[\sqrt{\rho_0}, V]^2 = 2\,\mathrm{tr}\,\rho_0 V^2 - \underbrace{2\,\mathrm{tr}\,\sqrt{\rho_0}\,V\,\sqrt{\rho_0} V}_{\text{hard to compute in nontrivial cases}} \leq 2\,\mathrm{tr}\,\rho_0 V^2$$

Modified MDS QSL: $\quad D_{\mathrm{tr}}(\rho_0, \rho_t) \leq \sqrt{2\langle V^2\rangle_\beta}\; t$

Local perturbation

local perturbation: $\langle V^2 \rangle_\beta$ is finite in the thermodynamic limit

for finite-range interactions, if $\langle V^2 \rangle_\beta$ is finite, $\langle [H_0, V]^2 \rangle_\beta$ is also finite

T-QSL *vs* generic QSLs: local perturbation

$$\langle V^2\rangle_\beta = O(1)_{N\to\infty}$$

MT $\quad D_{\rm tr}(\rho_0,\rho_t) \le \Delta E\, t$

$$\Delta E \equiv \sqrt{\langle H^2\rangle - \langle H\rangle^2} \sim \sqrt{N}$$

MDS

$$D_{\rm tr}(\rho_0,\rho_t) \le \sqrt{2\langle V^2\rangle_\beta}\; t = O(1)_{N\to\infty}$$

ML $\quad D_{\rm tr}(\rho_0,\rho_t) \le \sqrt{2\,\overline{\overline{E}}\,t}$

$$\sqrt{\overline{\overline{E}}} \equiv \sqrt{\langle H\rangle - E_{\rm gs}} \sim \sqrt{N}$$

thermal

$$D_{\rm tr}(\rho_0,\rho_t) \le \sqrt{\beta t}\;\sqrt[4]{-2\,\langle [H_0,V]^2\rangle_\beta}$$
$$= O(1)_{N\to\infty}$$

Finitely disturbing perturbation

finitely disturbing perturbation: $\langle [H_0, V]^2 \rangle_\beta$ is finite in the thermodynamic limit

not every finitely disturbing perturbation is local

Example 1: spin-boson model

$$H_0 = \Omega\,\sigma^z + \frac{1}{\sqrt{N}}\,\sigma^x \sum_k g_k(a_k^\dagger + a_k) + \sum_k \omega_k\, a_k^\dagger a_k$$

$$V = \sum_k \delta\omega\, a_k^\dagger a_k$$

non-local, but finitely disturbing

MDS QSL: $D_{\mathrm{tr}}(\rho_0, \rho_t) \le \sqrt{2}\,\delta\omega\, t\, \overline{n}_\beta\, N$

T-QSL: $D_{\mathrm{tr}}(\rho_0, \rho_t) \le \sqrt{\delta\omega\, \widetilde{g}\, \beta\, t}\, \sqrt[4]{2(1 + 2\,\widetilde{n}_\beta)}$

$$\overline{n}_\beta \equiv \sum_k \langle a_k^\dagger a_k\rangle_\beta / N \qquad \widetilde{g}^2 \equiv \sum_k g_k^2/N \qquad \widetilde{n}_\beta \equiv \sum_k g_k^2 \langle a_k^\dagger a_k\rangle_\beta \Big/ \sum_k g_k^2$$

finite in the thermodynamic limit

Example 2: mobile impurity model

$$H_0 = H_\mathrm{f} + P^2/(2m) + H_{\mathrm{imp-f}}$$

impurity-fluid interaction term

Hamiltonian of a fluid

mobile impurity particle with mass *m*,
P – momentum of the impurity

H_f describes a fluid in a 1D box of length *L*, with particle number *N* and particle density *n=N/L*

$$V = FX$$

linear potential felt by the impurity,
X – coordinate of the impurity

Example 2: mobile impurity model

$$H_0 = H_{\mathrm{f}} + P^2/(2m) + H_{\mathrm{imp-f}}$$

$$V = FX$$

non-local, but finitely disturbing

MDS QSL: $D_{\mathrm{tr}}(\rho_0, \rho_t) \leq \sqrt{2/3}\, N F t / n$

T-QSL: $D_{\mathrm{tr}}(\rho_0, \rho_t) \leq \sqrt{\beta t} \sqrt{(F/m)\sqrt{2\langle P^2 \rangle_\beta}}$

finite in the thermodynamic limit

Performance of T-QSL *vs* general QSLs in the many-body setting: summary

	Mandelstam-Tamm	Margolus-Levitin	Mondal-Datta-Sazim	thermal
zero temperature	loose	loose	exact	exact
trivial perturbation	loose	loose	exact	exact
local perturbation	loose	loose	tight	tight
finitely disturbing nonlocal perturbation	loose	loose	loose	tight

Generalization: time-dependent perturbation

$$i\partial_t \rho_t = [H_0 + V_t, \rho_t]$$

$$\rho_0 = e^{-\beta H_0}/Z_0, \quad Z_0 = \mathrm{tr} e^{-\beta H_0}$$

$$\boxed{D_{\mathrm{tr}}(\rho_0, \rho_t) \leq \sqrt{\beta \int_0^t dt' \sqrt{-2\left\langle [H_0, V_{t'}]^2 \right\rangle_\beta}}}$$

Summary

- a new quantum speed limit for initially thermal states derived
- it explicitly exploits structure of the thermal state and depends on temperature
- can be dramatically tighter than generic QSLs in the many-body setting

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Thank you for your attention!