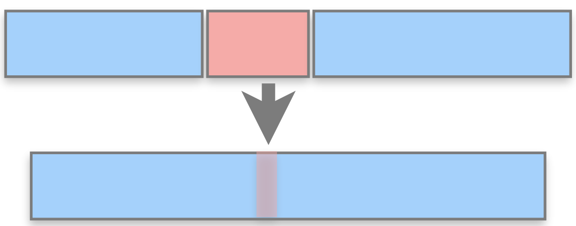
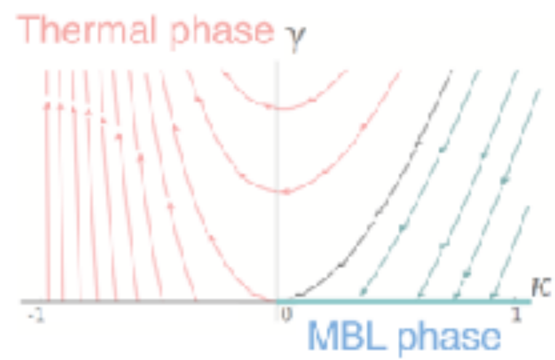
Kosterlitz-Thouless scaling at MBL phase transition



$\ell_{\text{new}} = \ell_1 + \alpha\, \ell_2 + \ell_3$

Maksym Serbyn

IST Austria



arXiv:1807.04285
arXiv: 1811.03103

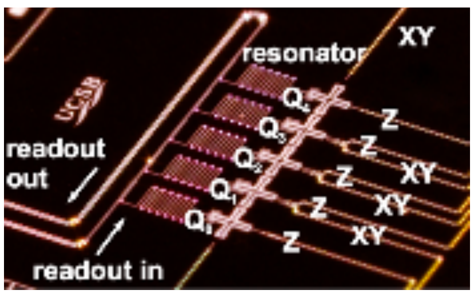
Moscow, 2018

collaboration with
A. Goremykina
R. Vasseur
P. Dumitrescu
S. Parameswaran

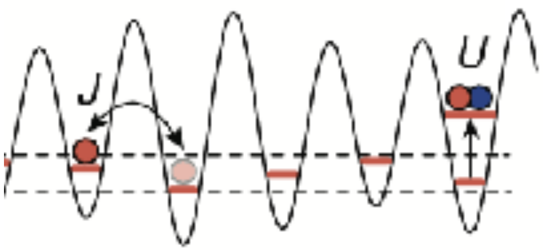
Universality in quantum dynamics

Artificial **isolated** quantum systems:

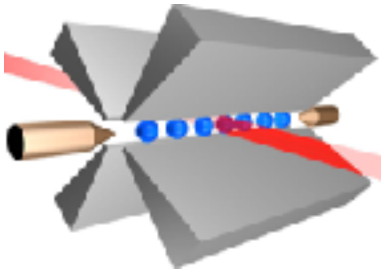
mutli-qubit systems



cold atoms



trapped ions



NV centers in diamond,

polar molecules,

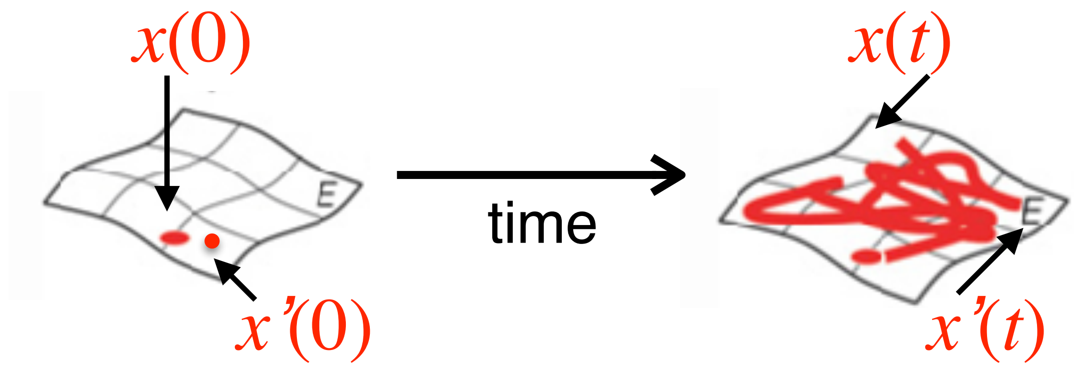
....

Generally, even perfectly isolated systems **thermalize**→ quantum information is destroyed

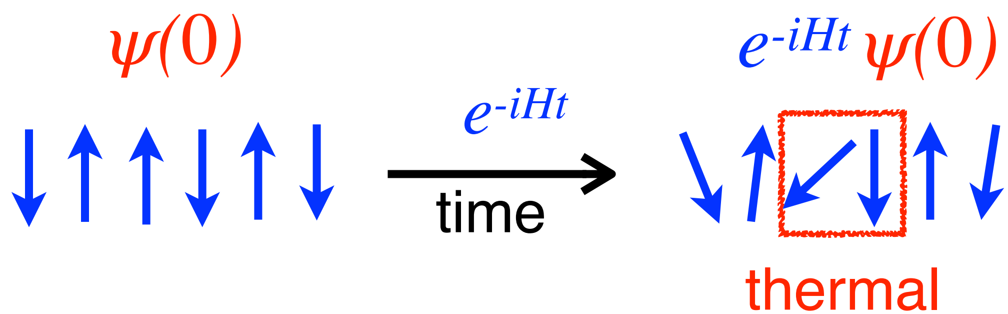
Thermalization in quantum systems

- Classical dynamics:

  chaos→ergodicity



- Quantum dynamics:



Eigenstate thermalization hypothesis:
eigenstates are thermal

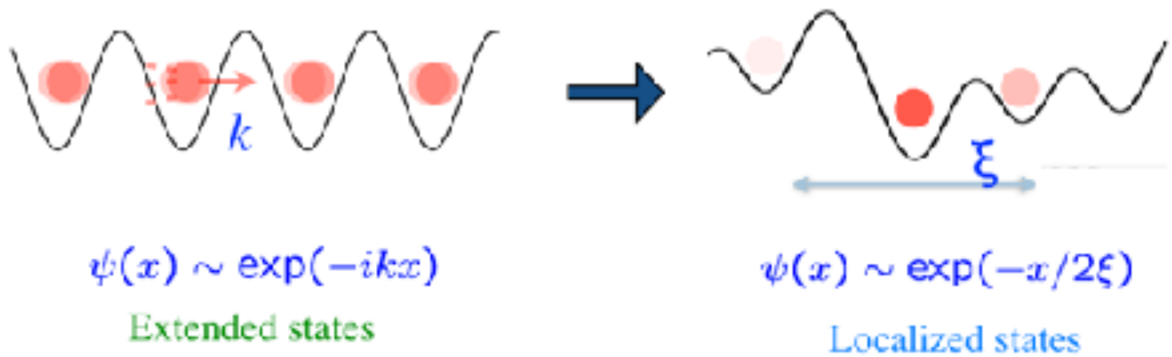
[Srednicki'94]
[Rigol,Dunjko,Olshanii'08]

- Prevent exchange of information to **avoid** thermalization

Escaping thermalization

- Non-interacting systems
- Bethe-ansatz integrability
- **Disorder**

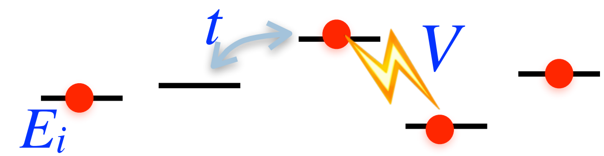
Anderson Insulator



P. W. Anderson

"an example of a real physical system with an infinite number of degrees of freedom, having no obvious oversimplification, in which the approach to equilibrium is simply impossible" [Anderson'58]

Many-Body Localized phase



- MBL = localized phase with interactions

  [Anderson, Fleishman'80]

  Limit of weak interactions: [Basko, Aleiner,Altshuler'05] [Gorniy, Polyakov,Mirlin'05]

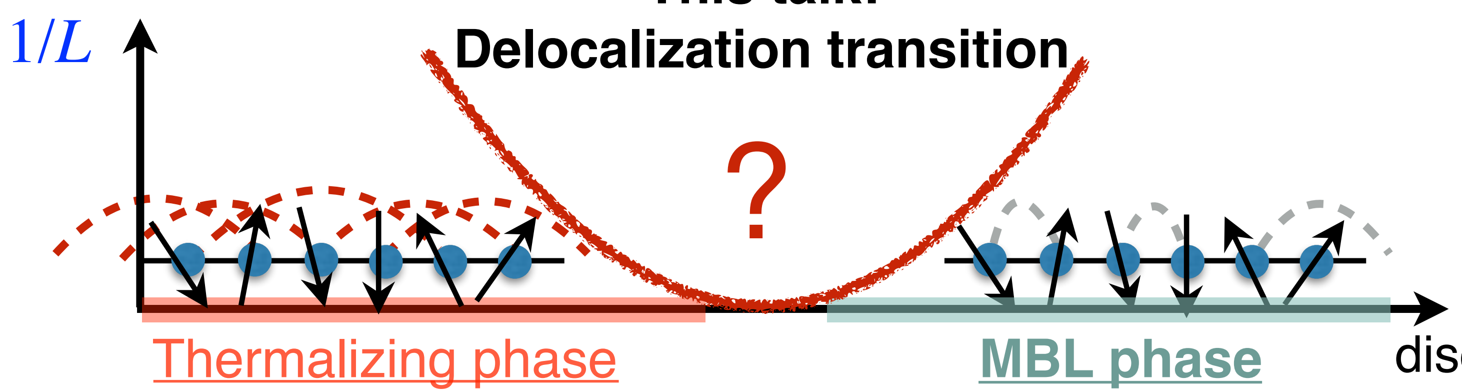
  Numerical evidence: [Oganesyan,Huse'08] [Znidaric,Prosen'08] [Pal,Huse'10]

  Phenomenology: LIOMS [MS,Papic,Abanin'13] [Huse,Oganesyan,Nandkishore'13]

  Experiments: MBL in 1d and 2d systems, entanglement growth, etc

  [Schreiber et al,'15] [Bordia et al,'16] [Choi et al'16] [Lukin et al,'18]

This talk:
Delocalization transition



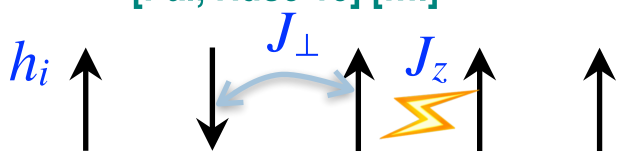
MBL transition

- New kind of a phase transition: nor classical nor quantum critical
  breakdown of thermalization

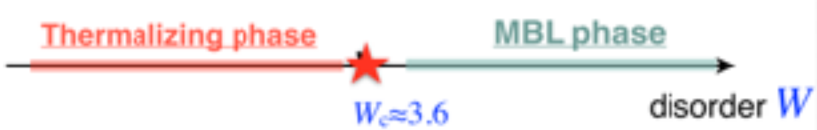
Microscopic approach

XXZ spin-1/2 chain

[Pal, Huse'10] [....]



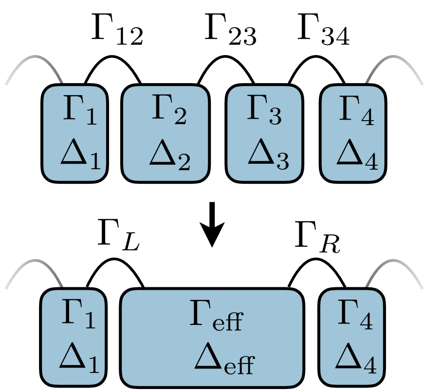
random $h_i \in [-W,W]$; interactions J_z



Numerics to study signatures

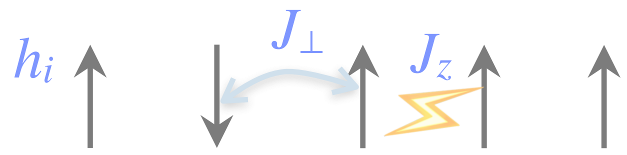
Phenomenological RG

Effective description:

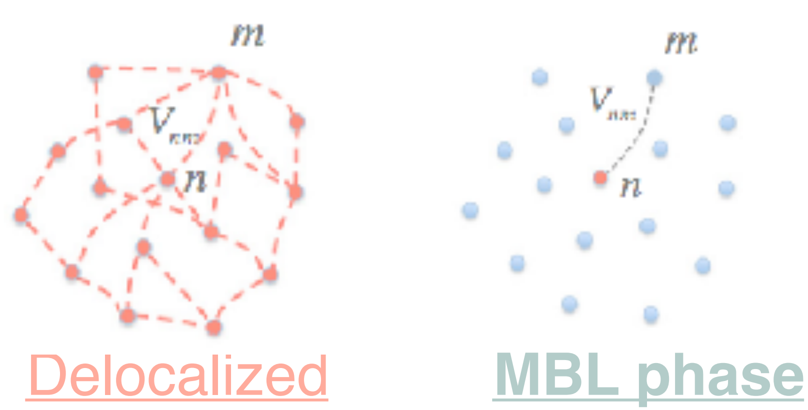


[Vosk,Huse,Altman '15]
[Potter,Vasseur&Parameswaran '15]

XXZ spin-1/2 chain

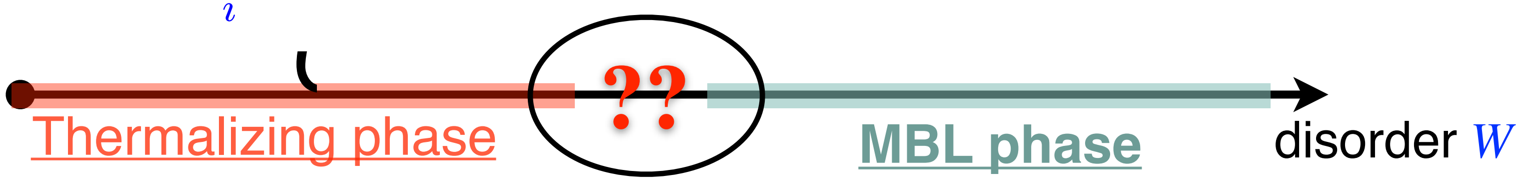


Microscopic models



Detecting delocalization transition in lattice models

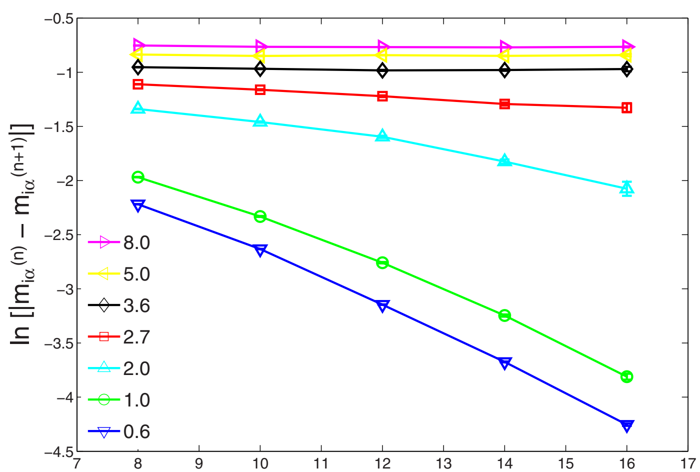
$$H = \sum_i \vec{S}_i \cdot \vec{S}_{i+1} + h_i S_i^z \qquad \text{random} \quad h_i \in [-W, W]$$



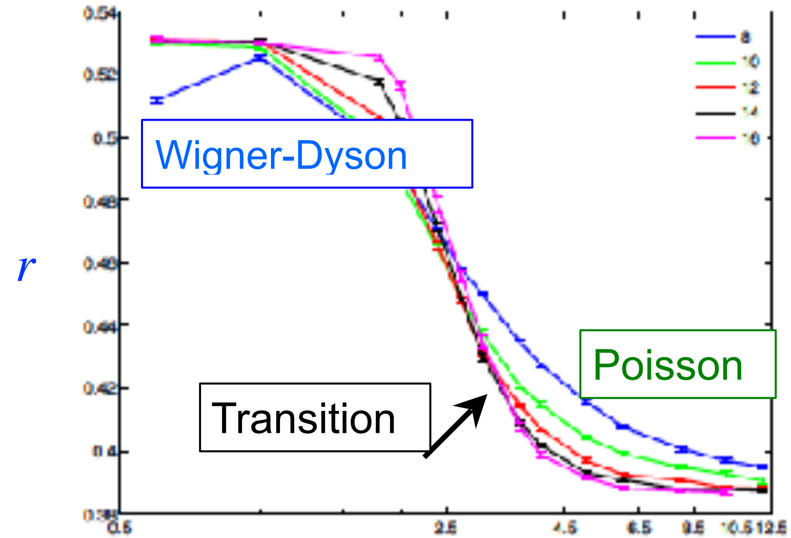
- Level statistics:

  [Oganesyan,Huse,PRB'07]
  [MS,Moore,PRB'16]

- ETH breakdown:



system size L

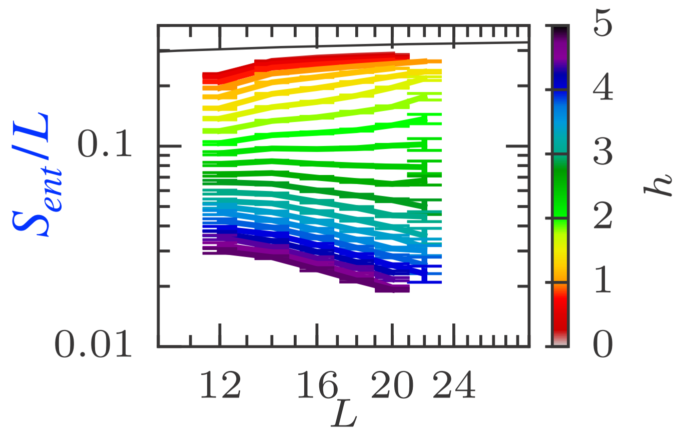


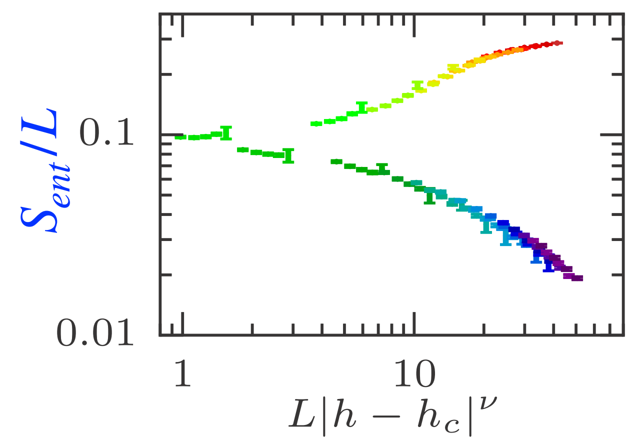
disorder W

[Pal and Huse, PRB'10]

Probes of MBL transition and scaling

- Extensive numerical study [Luitz et al, PRB'15]





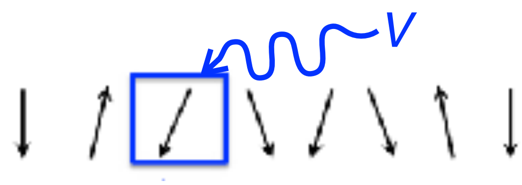
- Scaling collapse but unphysical exponent: $\nu \approx 0.8 \div 1.2 < 2$

- Alternatives?
  Scaling parameter for transition?

[MS, Papic, Abanin, PRX'15]
[Vosk, Huse, Altman, PRX'15]

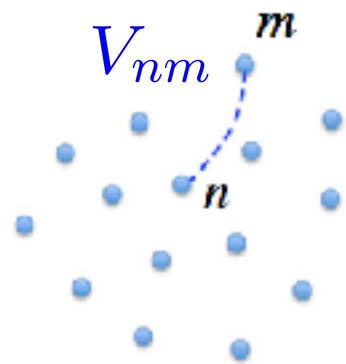
Effect of local operators in the eigenstate basis

- Effect of local perturbation on eigenstates:



$$H \to H + V$$

$$H|n\rangle = E_n|n\rangle \qquad (H+V)|\alpha\rangle = E_\alpha|\alpha\rangle$$



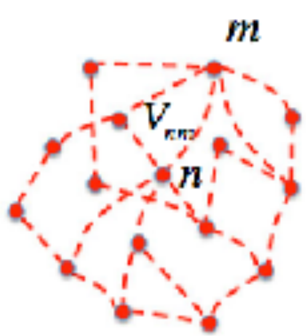
- New eigenstates are localized/delocalized?

- Parameter: $$\mathcal{G} = \log \frac{V_{i,i+1}}{E_i - E_{i+1}}$$

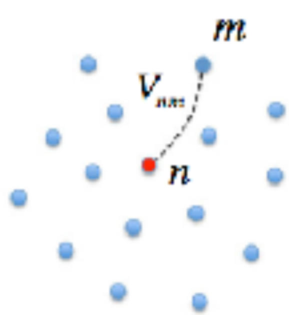
[MS, Papic, Abanin,PRX'15]

$\mathcal{G} \gg 1$

strong mixing
all spins perturbed



Delocalized



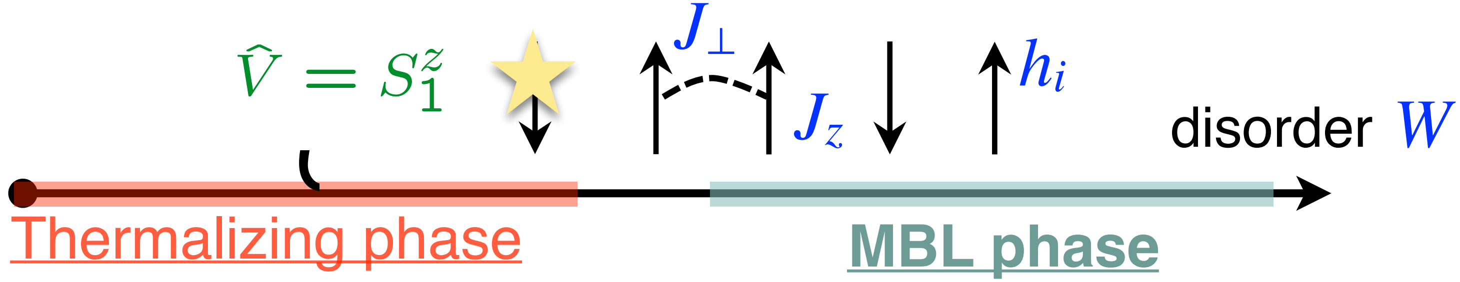
MBL phase

$\mathcal{G} \ll 1$

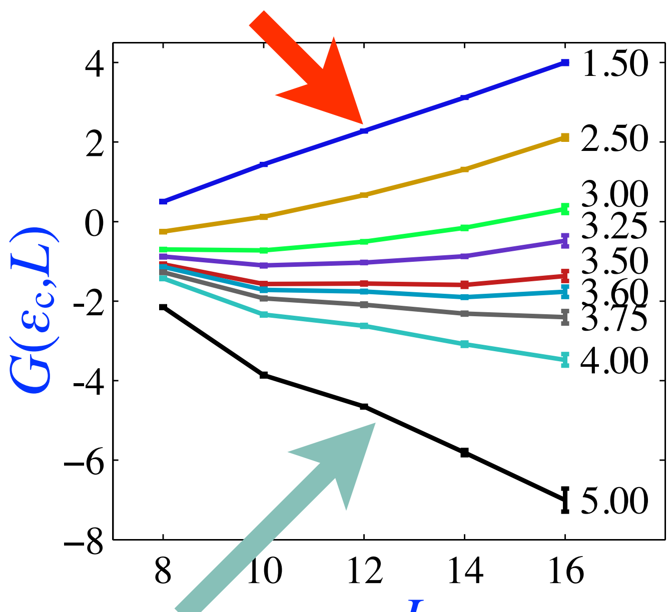
no resonances
τ are local

Distribution of Thouless conductance

- Scaling parameter: $\mathcal{G}(L) = \log \frac{V_{n,n+1}}{|E_n - E_{n+1}|}$
- Numerical results for XXZ spin chain: [MS, Papic, Abanin,PRX'15]

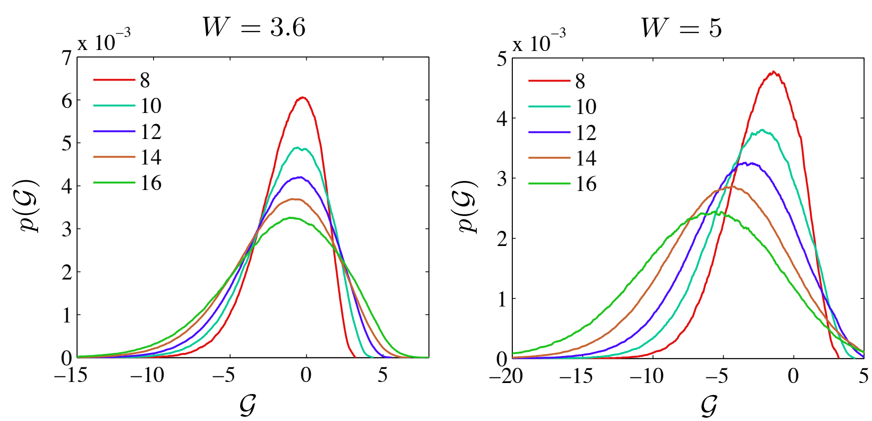


$G(\varepsilon,L) \propto +L \rightarrow$ delocalized



$G(\varepsilon,L) \propto -L \rightarrow$ MBL

But: **broad** distributions
@ transition/MBL phase

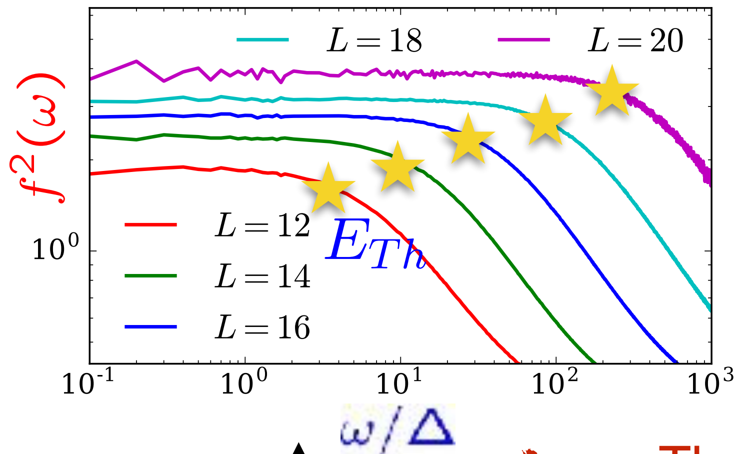


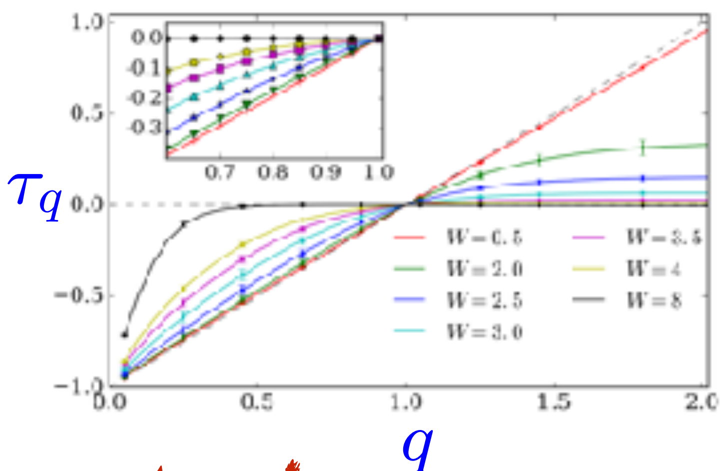
Broad critical region: Thouless energy & fractality

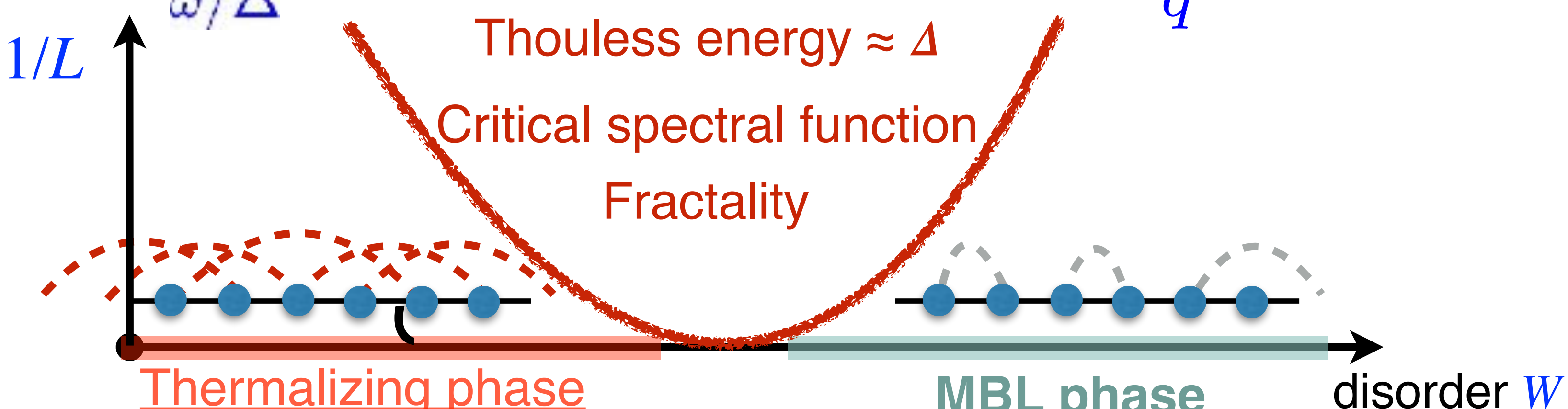
- Matrix elements as "wave function": [MS, Papic, Abanin,PRB'17]
  - ✳ Energy structure of matrix elements → 'Thouless energy'
  - ✳ Scaling dimension → 'fractality'

$$f^2(\omega) = e^{S(E)} \langle |\langle i|S^z|j\rangle|^2 \delta_{\omega, E_m - E_n} \rangle$$

$$P_q = \langle |\psi_m(n)|^{2q} \rangle \propto \frac{1}{\mathcal{D}^{\tau_q - 1}}$$





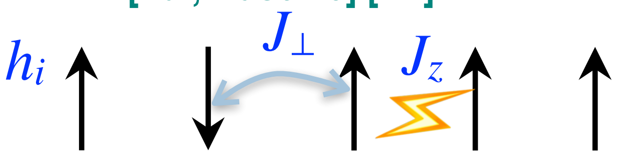


From microscopic to phenomenological approach

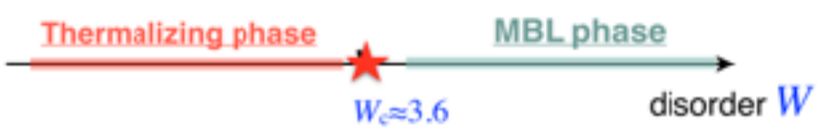
Microscopic approach

XXZ spin-1/2 chain

[Pal, Huse'10] [....]



random $h_i \in [-W, W]$; interactions J_z



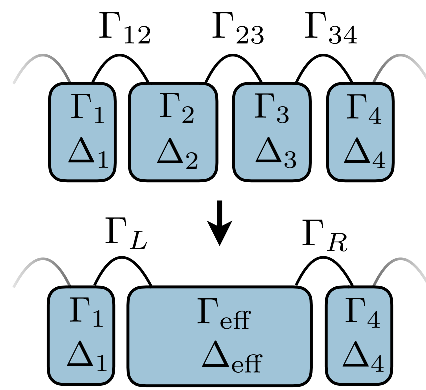
Numerics is consistent

but: unphysical critical exponents

& broad critical region

Phenomenological RG

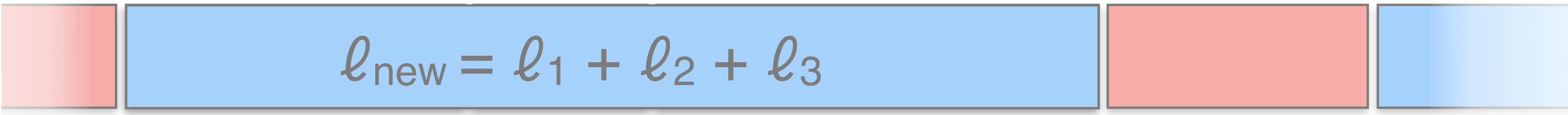
Effective description:



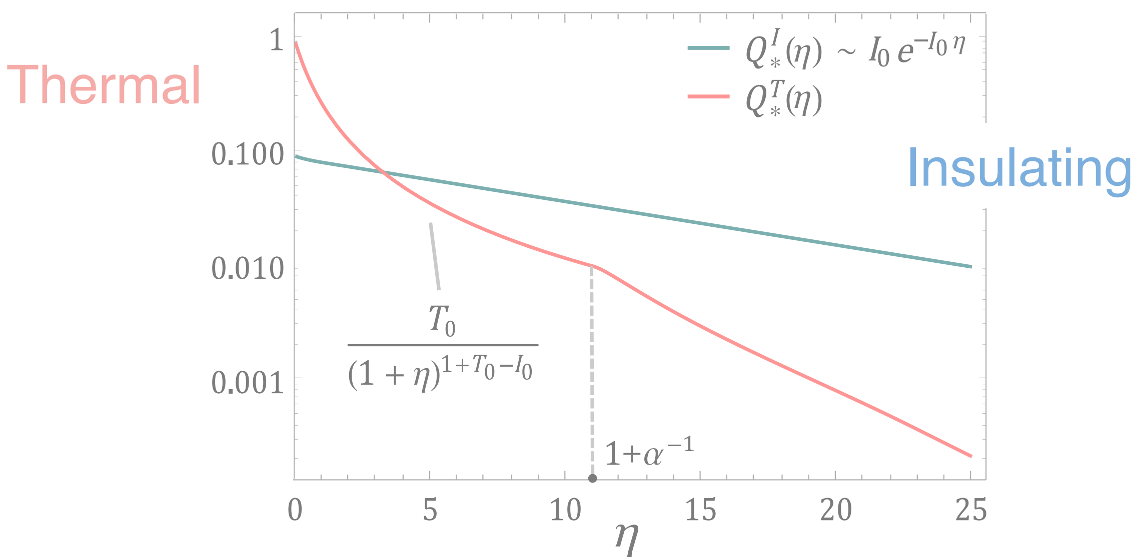
[Vosk,Huse,Altman '15]
[Potter,Vasseur&Parameswaran '15]
[Dumitrescu,Vasseur&Potter '17]
[Thiery,Muller,DeRoeck'17]

But: **numerics** to study signatures

Q: Analytical approach?

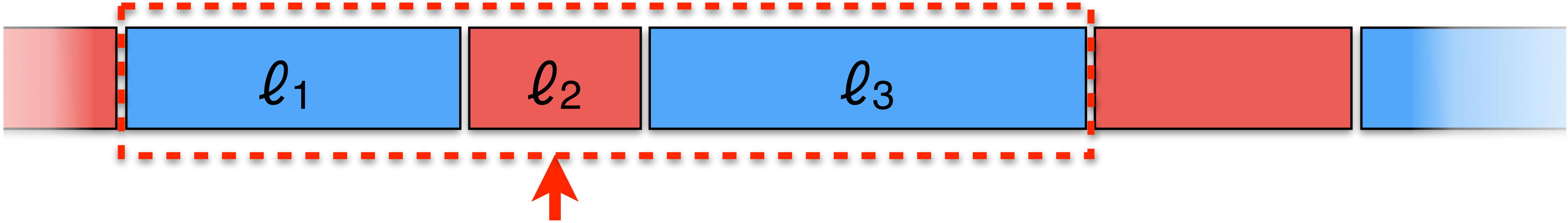


Analytically solvable RG

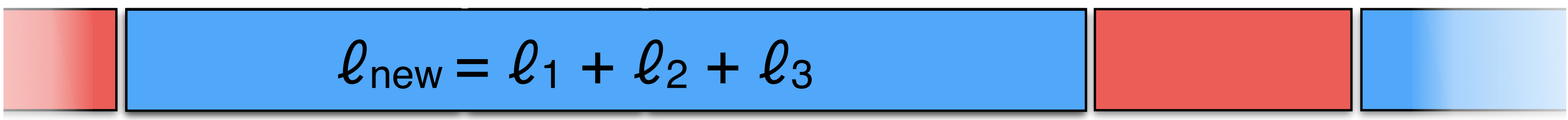


Oversimplified RG

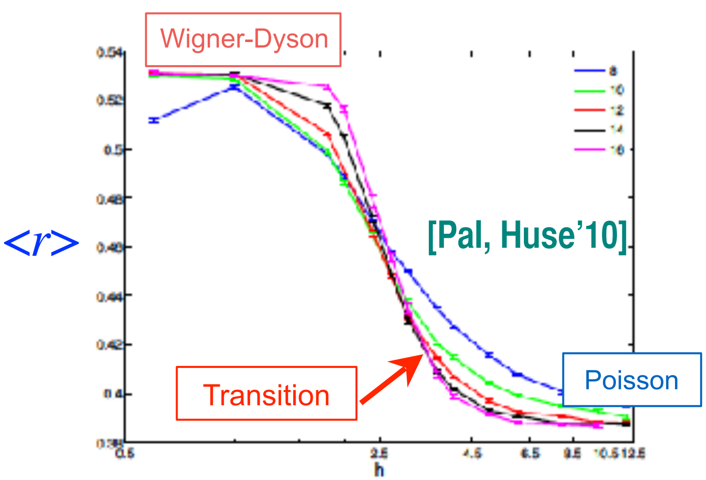
- Single parameter per block: ‘length’ [Zhang, Zhao, Devakul, Huse, PRB’16]



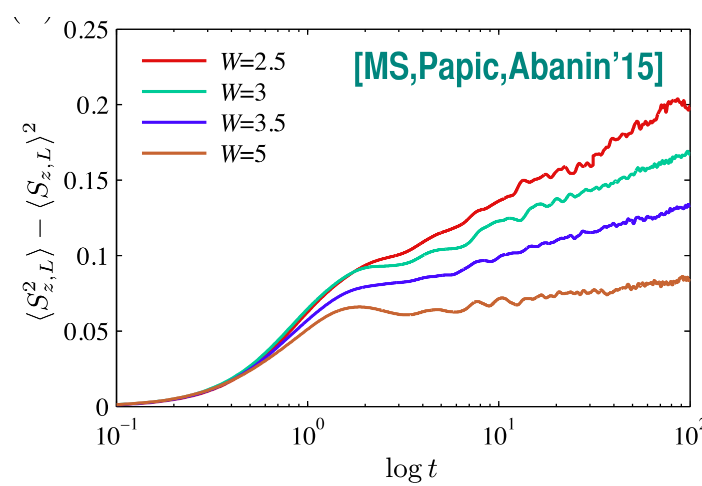
- At each RG step the shortest block is removed:



- Simple rules → analytic solution for fixed point [Bray, Derrida, PRE’95]
- Issue: **symmetry** between MBL and thermal phases, $p_{MBL} = p_{thermal} = 1/2$

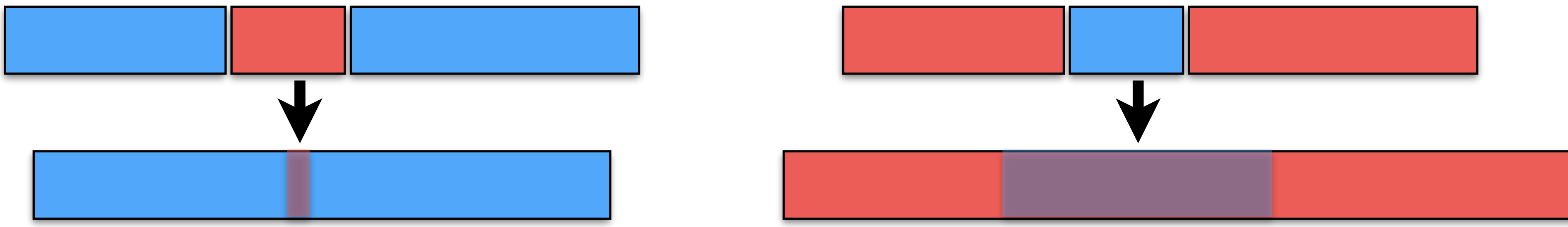


ED: transition point is **closer to MBL**



Oversimplified → simplified RG

- ‘Asymmetric’ coarsening problem:



$\ell_{\text{new}} = \ell_1 + \alpha\, \ell_2 + \ell_3$ $\qquad$ $\ell_{\text{new}} = \ell_1 + \beta\, \ell_2 + \ell_3$

- 2 parameter family of RGs, solution for any (α,β)
- Self-consistency constraint:

random AFM
spin chains
$\alpha=\beta=-1$ [Fisher PRB’94]

$$\text{Thermal } \ell_2 \xrightarrow{\alpha} \text{MBL } \alpha\ell_2 \xrightarrow{\beta} \text{Thermal } \alpha\beta\ell_2$$

$$\boxed{\alpha\beta = 1}$$

Conserved generalized total length: $\alpha\ell_{\text{tot}} + \ell_{\text{tot}}$

- α = ?… Use properties of transition to fix α

Anna
Goremykina

Coarse-grained equations and solutions

$\ell_{\text{new}} = \ell_1 + \alpha\, \ell_2 + \ell_3$ $\qquad$ $\ell_{\text{new}} = \ell_1 + \beta\, \ell_2 + \ell_3$

- Coarse grained description: distribution of thermal & insulating blocks

$$\eta = (\ell - \Gamma)/\Gamma \qquad \rho_\Gamma^{I,T}(\ell) = (1/\Gamma) Q_\Gamma^{I,T}(\eta)$$

- Flow with RG time $\Gamma = \min \ell$

$$\frac{\partial Q_\Gamma^I(\eta)}{\partial \ln \Gamma} = \partial_\eta \left[(1+\eta) Q_\Gamma^I(\eta)\right] + Q_\Gamma^I(\eta)[Q_\Gamma^I(0) - Q_\Gamma^T(0)] + Q_\Gamma^T(0)\theta(\eta-\alpha-1)\int_0^{\eta-\alpha-1} d\eta' Q_\Gamma^I(\eta') Q_\Gamma^I(\eta-\eta'-\alpha-1)$$

$$\frac{\partial Q_\Gamma^T(\eta)}{\partial \ln \Gamma} = \underbrace{\partial_\eta \left[(1+\eta) Q_\Gamma^T(\eta)\right]}_{\text{rescaling}} + \underbrace{Q_\Gamma^T(\eta)[Q_\Gamma^T(0) - Q_\Gamma^I(0)]}_{\text{block removal}} + \underbrace{Q_\Gamma^I(0)\theta(\eta-\beta-1)\int_0^{\eta-\beta-1} d\eta' Q_\Gamma^T(\eta') Q_\Gamma^T(\eta-\eta'-\beta-1)}_{\text{creation of a new block}}$$

- Stationary distributions $Q^{I,T}(\eta)$ at transition:
  - ✳ solve for $Q^{I,T}(\eta)$ → fixed point distributions
  - ✳ linearize around $Q^{I,T}(\eta)$ → critical exponent
  - ✳ fractality exponents → internal structure of blocks

[Zhang, Zhao, Devakul, Huse, PRB'16]

[Goremykina, Vasseur, MS, arXiv:1807.04285]

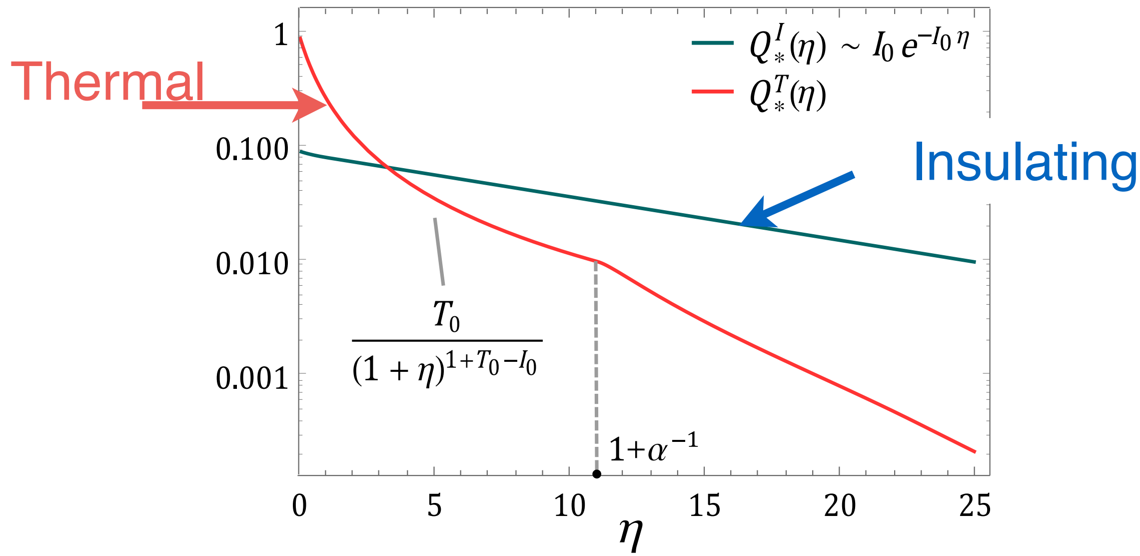
Physical properties on length-preserving line $\alpha\beta$=1

[Goremykina, Vasseur, MS, arXiv:1807.04285]

- Fixed point distributions:

  Convention: α<1, β>1

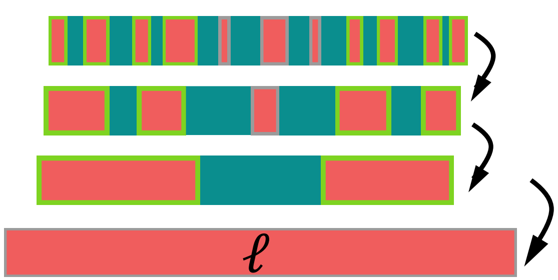
α=0.1 and β=10



- Probability to be thermal:

$$p_{\text{thermal}} = \frac{\alpha}{1+\alpha}$$

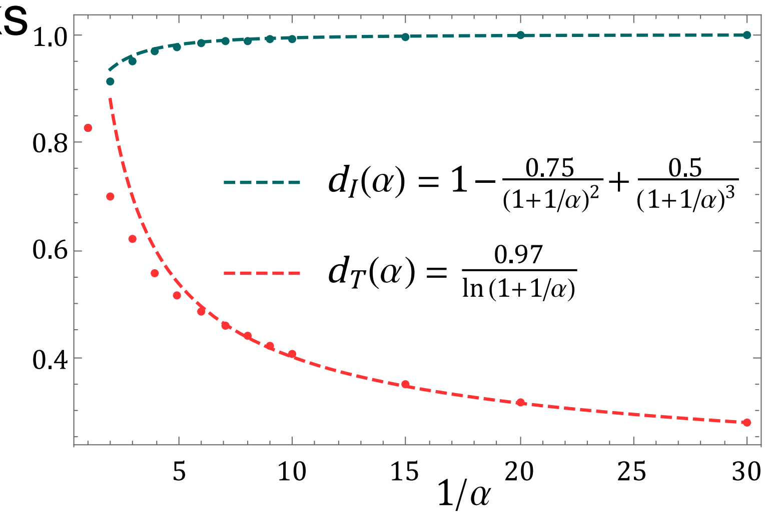
- Fractal exponent: internal structure of blocks



total length in the UV

$$\ell_f \equiv \Sigma \blacksquare \propto \ell^{d_T}$$

size of block in IR



- Limit $\alpha\to 0$: no fractal insulators $d_I \to 1$

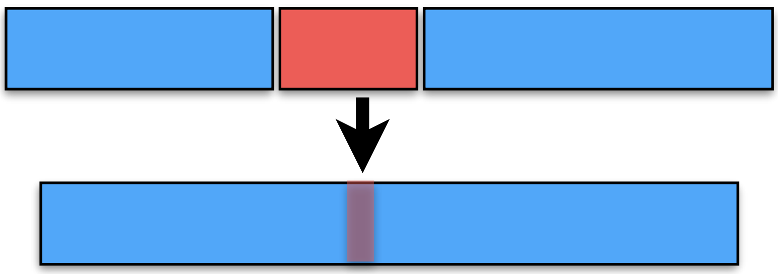
Physical justification of $\alpha \to 0$ limit

- Critical point is MBL with p → 1; no fractal insulators $d_I \to 1$
- Motivation of rules: 'length' ~ transport time; times add

$$\tau_T \sim \ell_T$$

$$\tau_I \sim \exp(\ell_I / \xi)$$

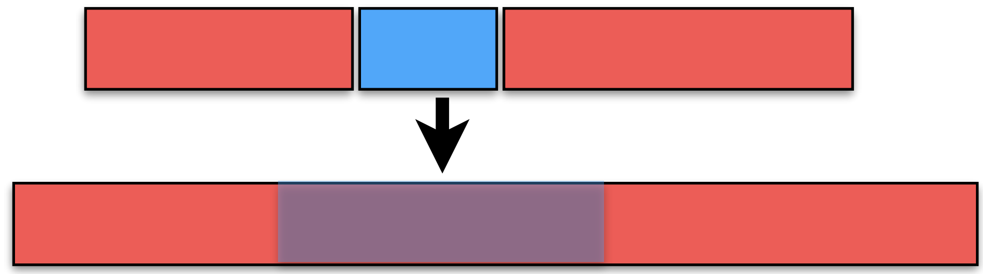
transport time ~ ℓ_2



$$\ell_{\mathrm{new}} = \ell_1 + \alpha\, \ell_2 + \ell_3$$

$$\alpha \to 0$$

transport time ~ $\exp(\ell_2/\xi)$



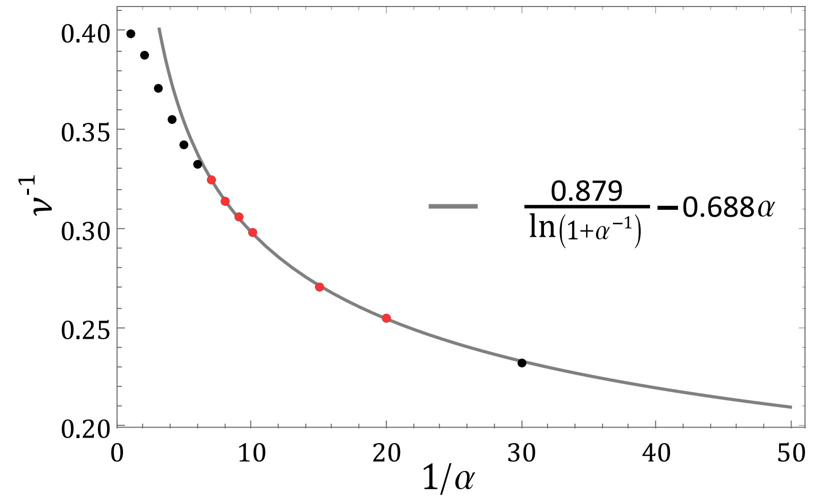
$$\ell_{\mathrm{new}} = \ell_1 + \beta\, \ell_2 + \ell_3$$

$$\beta \to \infty$$

- Limit $\alpha \to 0$ and $\beta \to \infty$ recovers same physics if $\alpha\beta$ is finite

[Goremykina,Vasseur,MS, in preparation]

Kosterlitz-Thouless flow

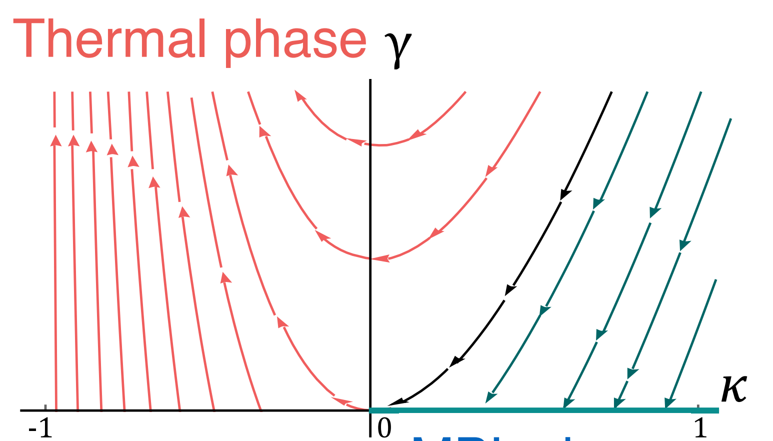


- Critical exponent ν diverges
- Higher order expansion of fRG eqs:

$$Q^I_\Gamma(\eta) = \gamma e^{-\gamma\eta}, \quad Q^T_\Gamma(\eta) = \frac{1+\kappa}{(1+\eta)^{2+\kappa}}$$

- Flow of γ, κ with cutoff:

$$\Gamma\frac{d\gamma}{d\Gamma} = -\gamma\kappa, \qquad \Gamma\frac{d\kappa}{d\Gamma} = -\gamma(1+\kappa).$$



- Implications:
  - ✳ KT-like flow
  - ✳ Power-law distribution of $Q^T(\eta)$ in MBL phase
  - ✳ MBL transition: <ℓ> diverges

[Goremykina, Vasseur, MS, arXiv:1807.04285]

Predictions from KT picture

- Critical exponent $\nu=\infty$;

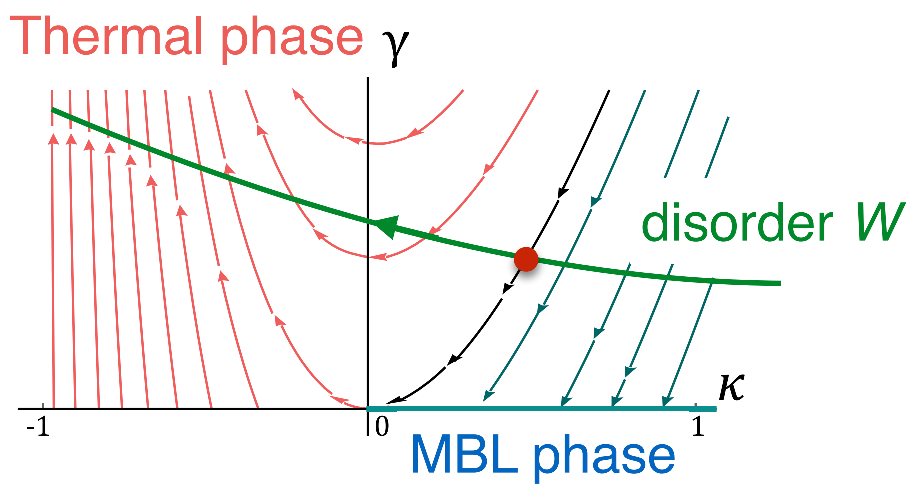
  correlation length diverges as

$$\xi \propto \exp(c/\sqrt{|W - W_c|})$$

- Algebraic distributions

  in MBL phase **&** at transition

$$Q^T(\eta) \propto \frac{1}{\eta^{2+\kappa}}$$



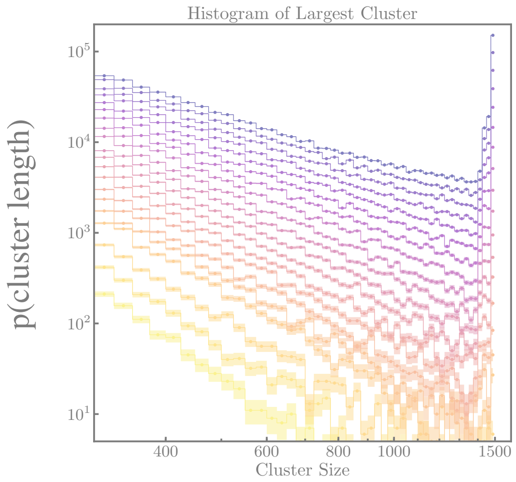
[Goremykina, Vasseur, MS, arXiv:1807.04285]

- Sparse thermal regions: fractal dimension $d_T \to 0$
- Overestimation of MBL transition
- **Q:** do other phenomenological RGs belong to KT universality class?

$$\frac{d\zeta^{-1}}{d\ln\Gamma} = -\rho\zeta^{-1}$$

$$\frac{d\rho}{d\ln\Gamma} = b\rho(1-\frac{\zeta}{\zeta_c})$$

Phenomenological approach and earlier RGs



Phenomenological approach to KT picture

[Dumitrescu et al, arXiv:1811.03103]

- Scaling variables: density of thermal regions ρ

  decay of matrix elements: $M(x) \sim e^{-x/\zeta}$

- Phenomenological flow:

  ζ always increases

$$\frac{d\zeta^{-1}}{d\ln\Gamma} = -\rho\zeta^{-1}$$

[Thiery,Huveneers,Müller &De Roeck, arXiv:1706.09338]

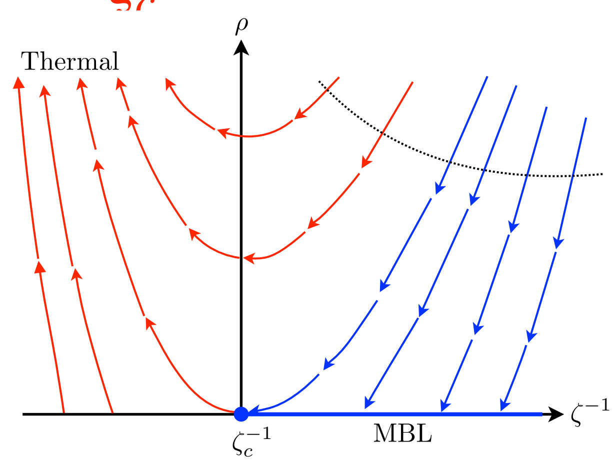
  ρ decreases/grows if $\zeta \gtrless \zeta_c$

$$\frac{d\rho}{d\ln\Gamma} = b\rho(1 - \frac{\zeta}{\zeta_c})$$

- Reduces to earlier picture if

$$\kappa = \frac{\zeta_c}{\zeta} - 1 \quad \text{and} \quad \gamma = \rho$$

- Q1: are other RGs in this universality class?



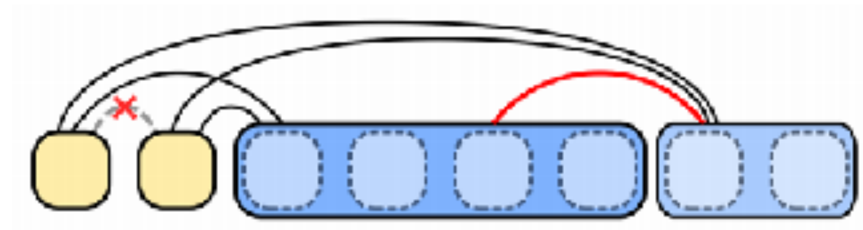
[Vosk,Huse,Altman,PRX'15]
[Dumitrescu,Vasseur&Potter, PRL 2017]

[Thiery,Huveneers,Müller& De Roeck, arXiv:1706.09338]

Algebraic correlations in cluster RG

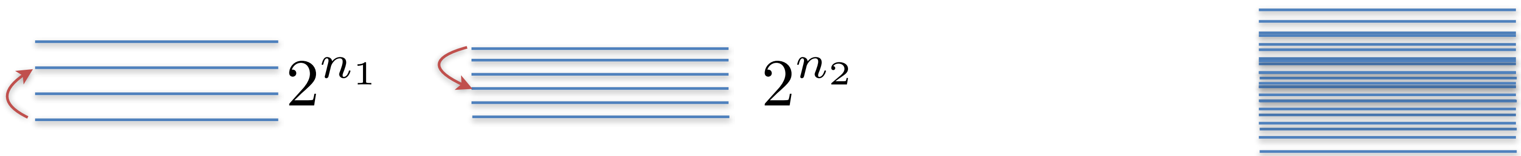
- Cluster RG

  [Potter,Vasseur&Parameswaran, PRX 2015]
  [Dumitrescu,Vasseur&Potter, PRL 2017]



“Merging” Resonant Clusters:

ignore substructure



2^{n_1} 2^{n_2} $2^{n_1+n_2}$

n_1 $\xleftrightarrow{x_{12}}$ n_2 $\Rightarrow$ $n_1 + n_2$

$$\Gamma(x_{12}) \gtrsim 2^{-(n_1+n_2)}$$

- All-to-all couplings: L × L matrix as variable

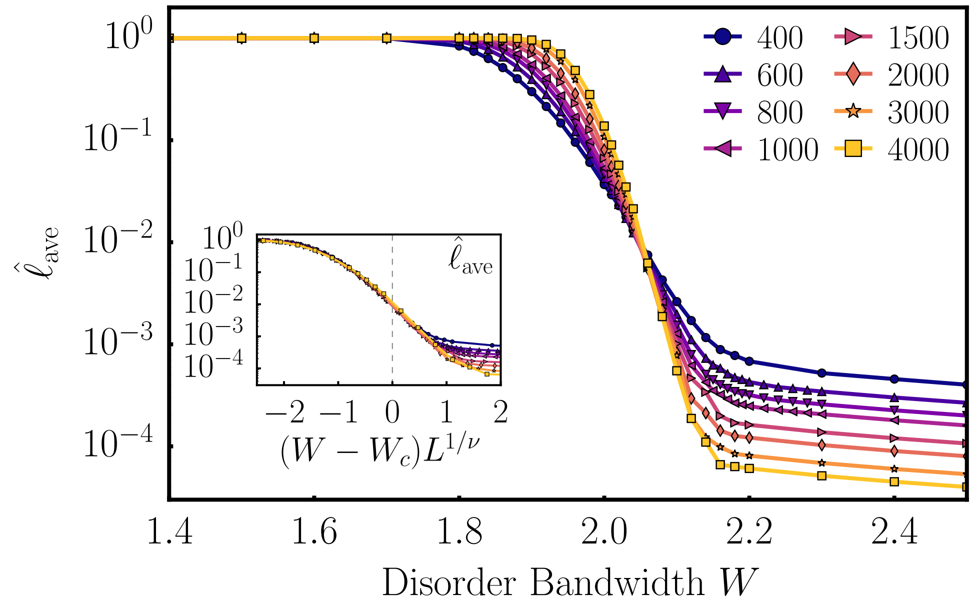
- RG is run numerically

- Universality class of this RG?

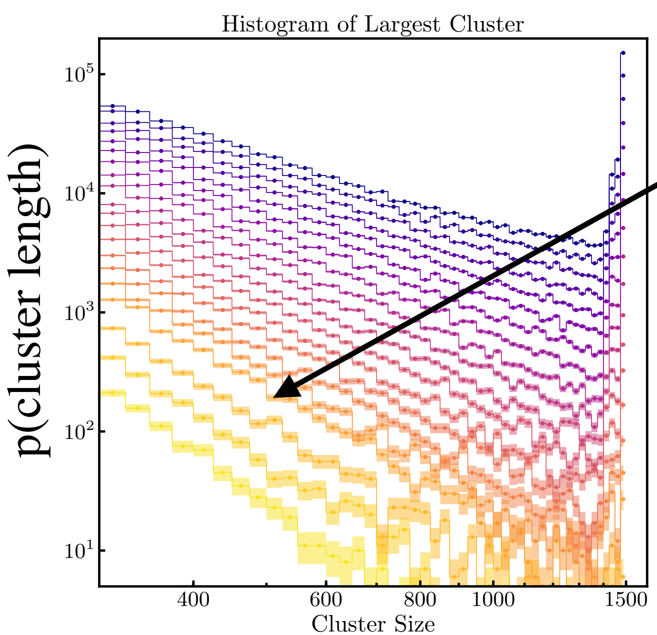
Revisiting cluster RG: KT picture

- Scaling collapse:
  finite exponent ν=3.2

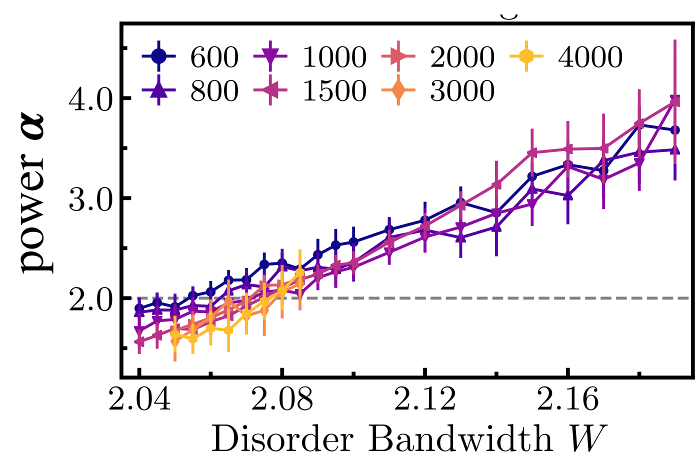
[Dumitrescu,Vasseur&Potter, PRL 2017]



- Revisiting in view of KT picture



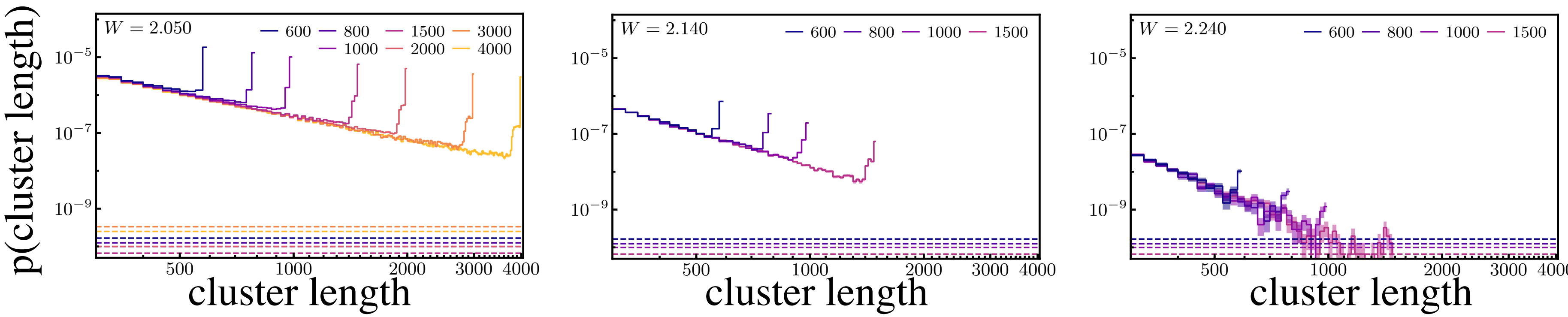
$$p(l) \sim \frac{1}{l^\alpha}$$



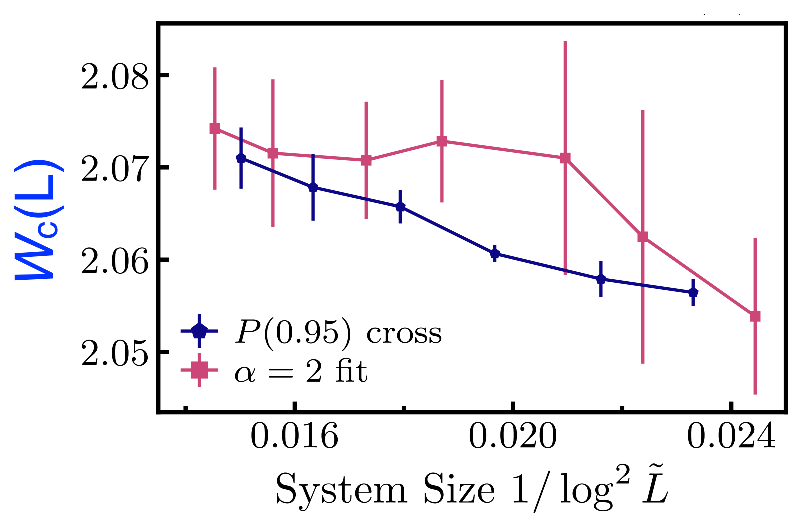
[Dumitrescu et al., arXiv:1811.03103]

Support for KT: power-law distributions

- Absence of cutoff even deep in the MBL phase (W_c=2.08)



- Finite size effects scaling:



- Everything is consistent with KT

[Dumitrescu et al, arXiv:1811.03103]

Outlook

- Analytically solvable family of RGs:

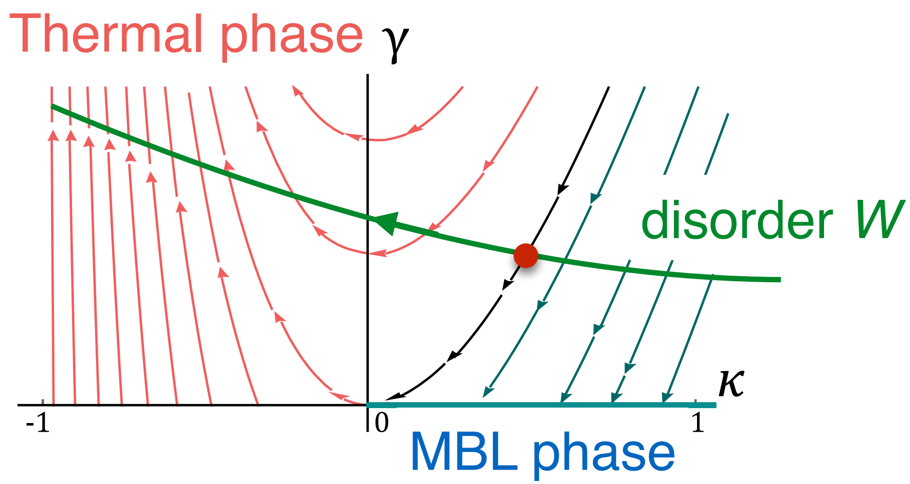
$$\ell_{\rm new} = \ell_1 + \alpha\, \ell_2 + \ell_3$$
$$\ell_{\rm new} = \ell_1 + \beta\, \ell_2 + \ell_3$$

- Kosterlitz-Thouless universality class:

  thermal segments in MBL phase:

$$Q^T(\eta) \propto \frac{1}{\eta^{2+\kappa}}$$

[Goremykina, Vasseur, MS, arXiv:1807.04285]



- Supported by phenomenological RG

  [Dumitrescu et al., arXiv:1811.03103]

- Open questions:
  - ✳ universality class of other phenomenological RGs?
  - ✳ connection to microscopic models

[Herviou, Bera, Bardarson, arXiv:1811.01925]

Acknowledgments

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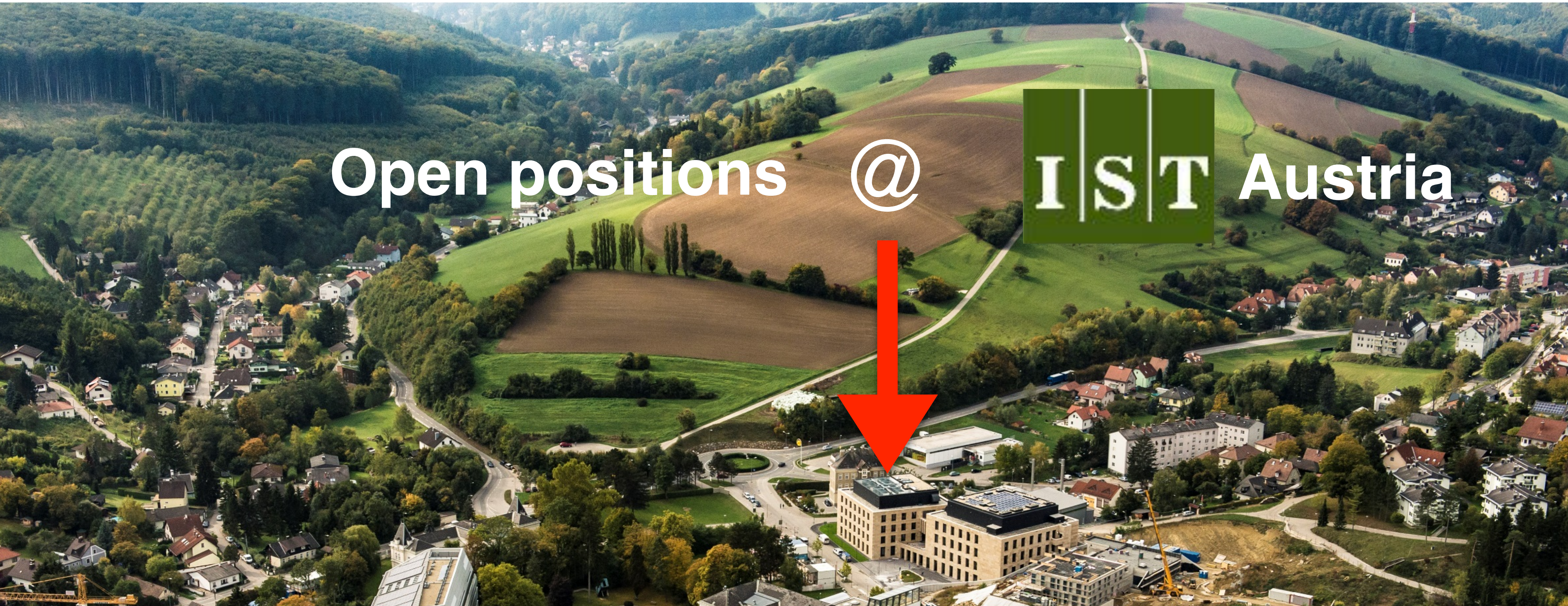

Outlook

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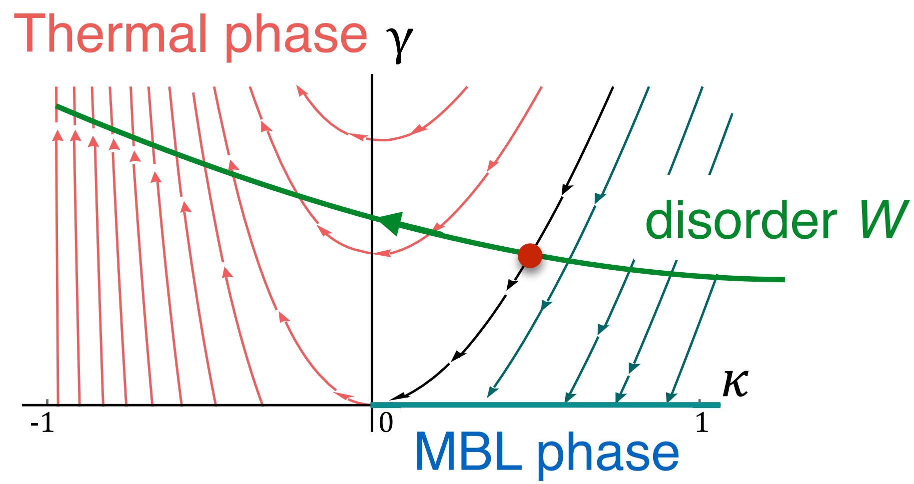

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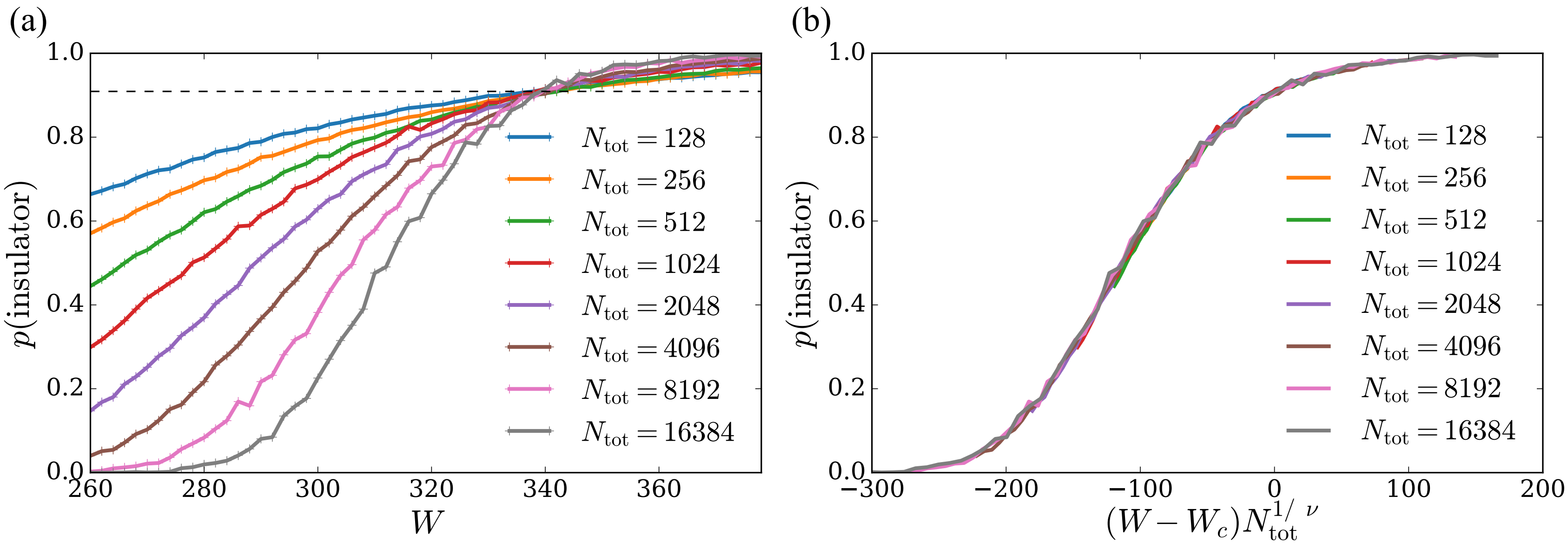
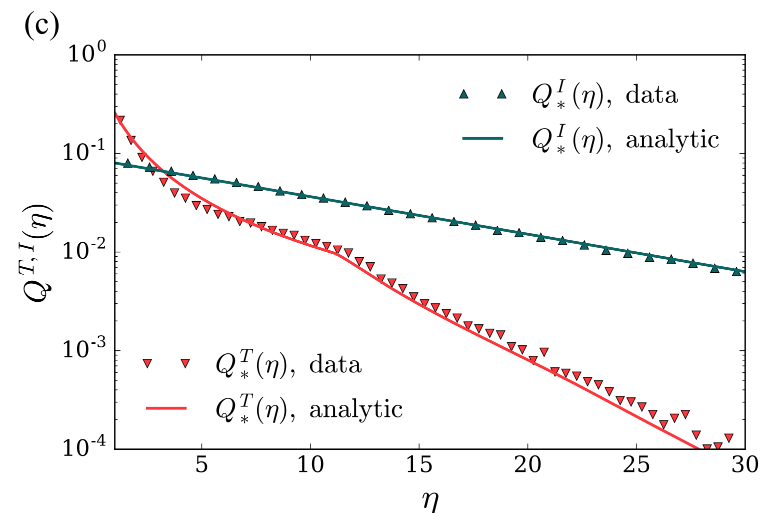
Appendix

Numerical checks of analytic solution

- Numerical implementation: transition for α=1/10, β=10



- Distributions at the critical point (no fitting parameters)



Critical exponent calculation

Critical exponent ν

Away from the fixed point:

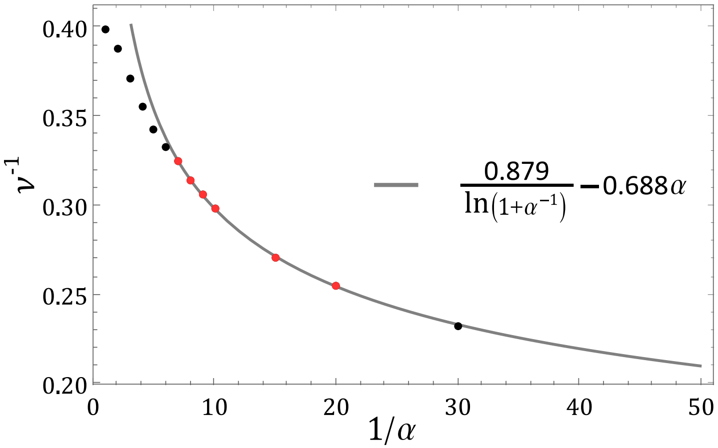
$$Q_{\Gamma}^{I,T}(\eta) = Q_{*}^{I,T}(\eta) + \Gamma^{1/\nu} f^{I,T}(\eta),$$

with $\int_0^\infty d\eta\, f^{I,T}(\eta) = 0$. Relevant perturbation: $\nu > 0$.

- Linearizing Eqs. (1), we obtain an eigenvalue system, $(1/\nu) f^{I,T}(\eta) = \hat{O}_{I,T} f^{I,T}(\eta)$, where $\hat{O}_{I,T}$ is a certain linear integro-differential operator.
- Numerical extrapolation suggests $1/\log(1+\beta)$ decay of $1/\nu$ with $\beta = 1/\alpha$.
- The eigenmodes in the limit $\alpha \to 0$

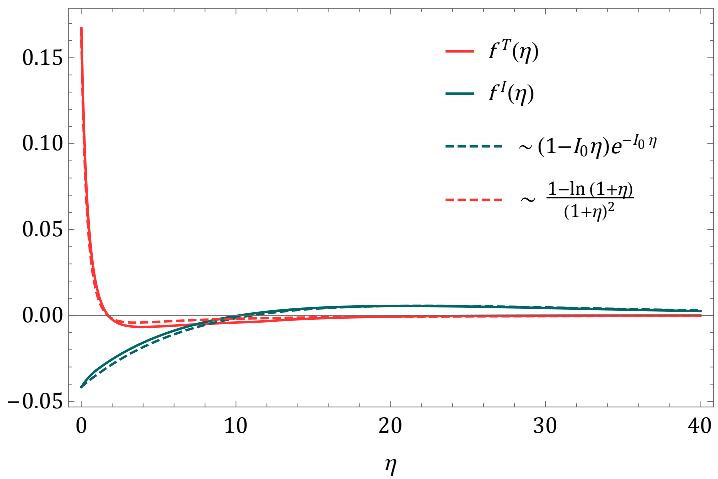
$$f^I(\eta) \sim (1 - I_0\eta)e^{-I_0\eta}, \quad f^T(\eta) \sim \frac{1-\log(1+\eta)}{(1+\eta)^2}$$

are well normalized and describe closely the eigenmodes for finite but large β.



Marginal critical exponent requires going to higher orders of perturbation theory $\Rightarrow$ we come up instead with an ansatz (2), describing the Kosterlitz-Thouless type flow!

Eigenmodes corresponding to leading eigenvalue



Cluster RG

- New numerical data: power-law distributions even in MBL phase